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RECENT
MILITARY, NAVAL, AND CIVIL SERVICE
EXAMINATION PAPERS
IN
MATHEMATICS,
WITH
ANSWERS AND HINTS FOR SOLUTION.

BY

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ST. JOHN'S COLLEGE, CAMBRIDGE.

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MILITARY, NAVAL, AND CIVIL SERVICE EXAMINATION PAPERS IN MATHEMATICS.

EXAMINATION FOR FIRST APPOINTMENTS TO THE CAVALRY AND INFANTRY.

May, 1873.

EUCLID. (BOOK I.) 1½ hr.

1. If one side of a triangle be produced, the exterior angle is greater than either of the interior opposite angles.

2. If two triangles have two angles of the one equal to two angles of the other, each to each, and one side equal to one side, viz., the side adjacent to the equal angles in each, then shall the other sides be equal, each to each, and also the third angle of the one equal to the third angle of the other.

3. Describe a parallelogram that shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

4. If the square described upon one of the sides of a triangle, be equal to the squares described upon the other two sides of it; the angle contained by these two sides is a right angle.

5. Describe a square on a given straight line as a diagonal.

6. Show that the sides of any four-sided rectilineal figure are together greater than the two diagonals.

Arithmetic. $2\frac{1}{2}$ hrs.

1. Add together $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$.

2. Subtract $2\frac{1}{4}$ from $20\frac{1}{8}$.

3. Multiply $\frac{4}{11}$ by $1\frac{4}{7}$.

4. Divide $2\frac{3}{4}$ by $1\frac{1}{16}$.

5. Add together 16·41215, 9·376, ·00403, and 270·3.

6. Subtract 17·2398 from 27·06.

7. Multiply 46·2375 by ·0074.

8. Divide 92·3784 by ·623 to three places of decimals.

9. Find the value of 1·25 of £1. 13*s.* 4*d.*

10. Reduce 4 tons 3 cwt. 1 qr. 9 lbs. to ounces.

11. Find the income-tax on £356. 10*s.* at 5*d.* in the £.

12. Find the simple interest on £576 for 6 years at $4\frac{1}{2}$ per cent. per annum.

13. Add together $1\frac{7}{12}$, $3\frac{7}{24}$, and $4\frac{1}{3}$.

14. Subtract $5\frac{3}{20}$ from $10\frac{1}{19}$.

15. Multiply together $\frac{9}{14}$, $\frac{5}{27}$, $9\frac{4}{5}$, and $2\frac{2}{7}$.

16. Divide $2\frac{13}{14}$ by $4\frac{1}{7}$.

17. Add together 2·6 of a day and ·85 of an hour, and give the answer in minutes.

18. Subtract 4·42 of a cwt. from 3·64 of a ton.

19. Multiply 62·5 by ·579.

20. Divide ·473928 by 24·18.

21. Express ·0425 of a mile in yards.

22. In 156704 square inches how many square yards and square feet are there?

23. What is the income of a man whose income-tax at 4*d.* in the pound amounts to 23 guineas?

24. At what rate per cent. will £230. 15*s.* amount to £305. 14*s.* 10½*d.* in 13 years at simple interest?

25. Find the dividend on £274. 10*s.* at 8*s.* 3*d.* in the £.

26. If 2 horses can plough 7 acres of ground in a day, how many horses will be required to plough 161 acres in 11½ days?

MATHEMATICS.

(ALGEBRA, LOGARITHMS, AND MENSURATION). 3 hrs.

1. Find the numerical value of $(x^2 - 2ax + a^2)$ when $x = 1$ and $a = \frac{1}{2}$; find also the value of x, if $(10000)^{\frac{1}{x}} = 10$.

2. Perform the operations indicated in the following examples:

(A) $\{2(a+b) - 3(c-d)\} - \{2(a-b) + 3(c+d)\}$.

(B) $(a^5 - a^4b + ab^4 - b^5) \times (a+b)$.

(C) $(16x^4 - 72x^2a^2 + 81a^4) \div (4x^2 + 12ax + 9a^2)$.

(D) $\dfrac{a^4 - a^2b^2 + b^4}{a^2 - ab\sqrt{(3)} + b^2}$.

(E) $\sqrt{(1 - 4x^{\frac{1}{2}} + 10x - 12x^{\frac{3}{2}} + 9x^2)}$.

3. Find the least common multiple of $(1-x)$, $(1-x^2)^2$, $(1-x)^3$.

Reduce to its lowest terms

$$\frac{3x^3 - 27ax^2 + 78a^2x - 72a^3}{2x^3 + 10ax^2 - 4a^2x - 48a^3}.$$

4. If $3x = \sqrt[3]{(9)}$, what is x?

Prove $$\frac{\sqrt[3]{(9)} - \sqrt[3]{(6)} + \sqrt[3]{(4)}}{\sqrt[3]{(9)} + \sqrt[3]{(6)} + \sqrt[3]{(4)}} \times \frac{3^{\frac{1}{3}} + 2^{\frac{1}{3}}}{3^{\frac{1}{3}} - 2^{\frac{1}{3}}} = 5.$$

5. Solve the following simple equations:

(A) $$\frac{x-2}{3} - \frac{x-4}{6} = 7 - \frac{x+3}{4}.$$

(B) $$\left.\begin{aligned} x + y + z &= 36 \\ x + 3y - 2z &= 12 \\ x - \frac{y}{3} + \frac{z}{2} &= 20 \end{aligned}\right\}.$$

6. A boat's crew row 9 miles with the tide in $\frac{3}{4}$ of an hour, and when the tide is flowing at half its former rate, the same crew row 9 miles against the tide in an hour and a half. Required the rate of the strongest tide and the rate at which the crew will row in still water.

7. Solve the following quadratic equations:

(A) $(x+3)^2 = 6x + 58$.

(B) $$\frac{3x-1}{2x} - \frac{2x}{3x-1} = \frac{24}{35}.$$

(C) $(x^3 - 8) = 4(x-2)(x+7)$.

8. If the lengths of two straight lines be each expressed in feet and inches, show how the product of their lengths may represent superficial measurement. Apply the rule of duodecimals to find the area of a room 32 feet 8 inches long by 18 feet 4 inches wide, and explain the different terms in the product.

9. Define the logarithm of a number. If the base were 3, of what numbers are 1, 2, 3, -1, -2, -3 respectively the logarithms? Why is it found most convenient to calculate logarithms to a base 10? Of what number is ·5 the logarithm to the base 10?

Find by the aid of the tables $\dfrac{(24{\cdot}76)^{\frac{2}{7}}}{({\cdot}0045)^{\frac{3}{2}}}$.

10 A right-angled triangle has a base of 240 feet, and the hypothenuse is 400 feet; find the area of the triangle.

11. A pyramid has a regular hexagon for its base, each side being 20 feet; find the cubical contents of the pyramid, if its altitude is 12 feet.

12. The three edges of a rectangular parallelepiped that meet in an angle are respectively 25, 54, 160. Find the side and diagonal of a cube which has the same volume as the parallelepiped.

13. A field has 5 sides; show by what measurements the field may be divided into triangles and its area found.

MATHEMATICS.

(GEOMETRY AND TRIGONOMETRY). 3 hrs.

1. If a straight line be divided into two equal parts, and also into two unequal parts; the rectangle contained by the unequal parts, together with the square of the line between the points of section, is equal to the square of half the line.

Illustrate the truth of this proposition by taking the whole straight line AB to be 12 feet, and the two parts AD and DB to be 9 feet and 3 feet respectively.

2. If a straight line drawn through the centre of a circle bisect a straight line in it which does not pass through the centre, it shall cut it at right angles.

A point is taken within a circle and from that point two equal straight lines are drawn to the circumference. Prove that the straight line which bisects the angle between these two straight lines passes through the centre of the circle.

3. If one circle touch another internally in any point, the straight line which joins their centres being produced shall pass through that point of contact.

4. Inscribe a circle in a given triangle.

If the points of contact be joined prove that each angle of the triangle so formed together with half the angle opposite to it in the original triangle is equal to a right angle.

5. Describe a circle about a given square.

Prove that the area of the square inscribed in any circle is half that of the square described about the same circle.

6. If a straight line be drawn parallel to one of the sides of a triangle, it shall cut the other sides, or these produced, proportionally.

7. Equal parallelograms, which have one angle of the one equal to one angle of the other, have their sides about the equal angles reciprocally proportional.

8. Prove that the ratio of the diameter of the circle described about a triangle to any one of the sides is the same as the ratio of half the rectangle of the remaining two sides of the triangle to the area of the triangle.

9. Define the grade and the degree, and express the interior angle of a regular pentagon both in grades and degrees.

10. Define the circular measure of an angle, and find two angles such that their difference is one degree, and their sum is the unit of circular measure.

11. Define the principal trigonometrical ratios, and find the sine of 30° and secant of 45°.

12. Prove that $\sin(A-B) = \sin A \cos B - \cos A \sin B$, stating what limitations your proof supposes in the values of A and B respectively.

13. Find $\sin 2A$ and $\cos 3A$.

14. If $\sin A = \dfrac{2ab}{a^2 + b^2}$ find $\tan \dfrac{A}{2}$.

15. Prove that the area of any triangle whose sides are a, b, and c is

$$\sqrt{\{s(s-a)(s-b)(s-c)\}},$$

s being the semi-perimeter.

16. In the triangle ABC

$$A = 60^\circ\ 15', \quad B = 54^\circ\ 30', \quad \text{and} \quad AB = 100 \text{ yards},$$

find AC having given

$$L \sin 54^\circ\ 30' = 9{\cdot}9106860,$$
$$L \sin 65^\circ\ 15' = 9{\cdot}9581543,$$
$$\log {\cdot}89646 = \bar{1}{\cdot}9525317.$$

17. The sides and the included angle of a triangle are 327 feet, 256 feet, and 56° 28′ respectively, find the remaining angles given

$$\log {\cdot}71 = \bar{1}{\cdot}8512583,$$
$$\log {\cdot}583 = \bar{1}{\cdot}7656686,$$
$$L \tan 61^\circ\ 46' = 10{\cdot}2700705,$$
$$L \tan 12^\circ\ 46' = 9{\cdot}3552267,$$
$$L \tan 12^\circ\ 47' = 9{\cdot}3558126.$$

18. ABC is a triangle and D the middle point of the base BC, prove that the sine of BAD is

$$\frac{b \sin A}{\sqrt{(b^2 + 2bc \cos A + c^2)}}.$$

EXAMINATION FOR FIRST APPOINTMENTS TO THE CAVALRY AND INFANTRY.

August, 1873.

EUCLID. (BOOK I.). $1\frac{1}{2}$ hr.

1. Any two sides of a triangle are together greater than the third side.

2. Construct a triangle of which the sides shall be equal to three given straight lines, but any two whatever of these must be greater than the third.

3. Parallelograms on the same base, and between the same parallels, are equal to one another.

4. To a given straight line apply a parallelogram, which shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

5. Define a rhombus, and show that its opposite angles are equal.

6. If a straight line be drawn through A, one of the angular points of a square, cutting one of the opposite sides, and meeting the other produced at F, show that AF is greater than the diagonal of the square.

ARITHMETIC. $2\frac{1}{2}$ hrs.

1. Add together $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{8}{9}$.
2. Subtract $3\frac{1}{6}$ from $10\frac{1}{9}$.
3. Multiply $\frac{3}{13}$ by $1\frac{5}{8}$.
4. Divide $3\frac{2}{3}$ by $1\frac{1}{15}$.

5. Add together 8·376, ·06703, 37·04, and 28·064015.

6. Subtract 28·453$\dot{2}$ from 320·6.

7. Multiply 4·2635 by ·068.

8. Divide 8·4605 by ·248 to three places of decimals.

9. Find the value of 3·75 of £2. 14*s.* 8*d.*

10. In 1250784 seconds how many weeks, days, &c. are there?

11. If 3 tons of iron cost £16. 10*s.*, what will be the value of 3 cwt. 2 qrs. 14 lbs.?

12. Find the simple interest on £732 for 8 years at $3\frac{1}{4}$ per cent. per annum.

13. Add together $1\frac{7}{10}$, $3\frac{7}{20}$, and $2\frac{1}{25}$.

14. Subtract $5\frac{3}{16}$ from $10\frac{1}{8}$.

15. Multiply together $1\frac{2}{9}$, $\frac{5}{12}$, $8\frac{1}{10}$ and $2\frac{1}{22}$.

16. Divide $2\frac{13}{16}$ by $4\frac{1}{8}$.

17. Add together 2·16 of a yard and ·05 of a mile.

18. Subtract 4·42 of an hour from 3·64 of a day.

19. Multiply ·506 by ·632.

20. Divide 13·915 by 575.

21. Express 13*s.* 4*d.* as the decimal of £4.

22. In 8156704 cubic inches, how many cubic yards and cubic feet are there?

23. When the income-tax was reduced from 6*d.* to 4*d.* in the pound, a person found that he had £42. 3*s.* 9*d.* less income-tax to pay. What was his income?

24. In what time will £230. 15*s.* amount to £305. 14*s.* $10\frac{1}{2}$*d.* at $2\frac{1}{2}$ per cent. simple interest?

25. Find the dividend on £2,560. 5*s.* at 12*s.* 8*d.* in the £.

26. If 7 men receive £17. 10*s.* wages for 15 days, what will be the wages of 30 men for 6 days?

MATHEMATICS. 3 hrs.

1. Multiply $x^3 - 3x^2 + 11x - 15$ by $x^2 - 5x + 3$, and find the value of the product when $x = 8$.

Resolve into factors the expressions

$$(\alpha)\ (5x + 4a)^2 - (4x + 5a)^2;$$

$$(\beta)\ x^2 - (a + b)\,x + ab;$$

and $$(\gamma)\ x^2 - 18x + 77.$$

2. Divide:

$$(\alpha)\ x^4 - x^3 + 3x^2 - 71x + 44 \text{ by } x - 4;$$

$$(\beta)\ x^3 + y^3 + z^3 - 3xyz \text{ by } x + y + z;$$

and $$(\gamma)\ x^4 - 7x^2y^2 + y^4 \text{ by } x^2 + 3xy + y^2.$$

3. Find the highest common factor and the least common multiple of the expressions

$$x^3 - 7x^2 + 33x - 63 \text{ and } x^3 - 6x^2 + 30x - 63.$$

Find the value of r when the two expressions $x^2 - 9x + 20$, and $x^2 - (r + 3)\,x + 3r$ have a common factor.

4. If $\frac{a}{b} = \frac{c}{d}$, prove that $\frac{a+b}{a-b} = \frac{c+d}{c-d}$, and that

$$\frac{3a^4 + 4b^4}{3a^4 - 4b^4} = \frac{3c^4 + 4d^4}{3c^4 - 4d^4}.$$

5. Find the square of $\sqrt{(3)} - 4\sqrt{(5)} + \sqrt{(15)}$, and the square root of $152 - 30\sqrt{(15)}$.

Calculate to four places of decimals the value of the expression

$$\frac{24\sqrt{(5)} - 5\sqrt{(3)}}{2\sqrt{(5)} - \sqrt{(3)}}.$$

6. Prove that the sum of the cubes of three consecutive numbers is divisible by the sum of the numbers, and that the difference between the fourth powers of the greatest and least of the three numbers is divisible by eight times the middle number.

7. Solve the following equations:

(α) $\frac{7x+4}{9} - \frac{4x+5}{7} + \frac{2}{5}(x+1) = \frac{3x+1}{5}$.

(β) $\frac{x^2+5}{x} = \frac{4x+5}{4}$.

(γ) $x^2 - 88x + 1911 = 0$.

(δ) $x^2 + 4\sqrt{(x^2+4)} = 41$.

(ε) $\left.\begin{aligned} x^2 + xy + y^2 &= 331 \\ x^2 - xy + y^2 &= 111 \end{aligned}\right\}$.

8. A number of two digits is such that, if the digits be inverted, the new number is less by 9 than the original number, and the sum of the squares of the digits is 113; find the number.

9. A body of 684 soldiers is to be drawn up in the form of a hollow square, three deep; find the number of men in the front of each side of the square.

10. Define the logarithm of a number to a given base, and find the logarithms of 10000 to the base 10 and to the base $\sqrt[3]{(10)}$, and also of 2401 to the base 7, and to the base $\frac{1}{\sqrt[3]{(7)}}$.

Find from the tables $\log_{10}(487{\cdot}9634)$, and, by help of the tables, find an approximate value of

$$(3{\cdot}7452)^{\frac{1}{3}} \div (3{\cdot}7121)^{\frac{1}{4}}.$$

11. The sides of a right-angled triangular field, adjacent to the right angle, are 357 feet and 476 feet; find the length of the third side of the field, and the number of square feet in its area.

12. The three different edges of a rectangular parallelopiped are respectively 3 feet, 2·52 feet, and 1·523 feet in length; find the number of cubic feet in the parallelopiped.

Find also the cubic space inside a box of the same external dimensions, constructed of a material one-tenth of a foot in thickness.

13. A river 20 feet deep and 100 yards wide is flowing at the rate of 3 miles an hour; find how many tons of water run into the sea per minute, it being given that a cubic foot of water weighs 1,000 ounces.

14. Assuming that the ratio of the perimeter of a circle to its diameter is that of 3·1416 to unity, find the volume and the surface of a cylinder, the radius of which is 4 feet and the height 100 feet.

Supposing that a wedge is cut out of this cylinder by two planes through its axis inclined to each other at an angle, which is half a right angle, what will be the volume left?

MATHEMATICS.

(GEOMETRY AND TRIGONOMETRY). 3 hrs.

1. If a straight line be bisected, and produced to any point, the rectangle contained by the whole line thus produced, and the part of it produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

2. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

Prove that the given straight line has also been divided so that the squares on the whole and one of the parts are together equal to three times the square on the other part.

3. Equal straight lines in a circle are equally distant from the centre.

If two parallel chords *AB*, *CD* be drawn in a circle then *AC* and *BD* are equally distant from the centre.

4. Describe a circle about a given triangle.

If the triangle be right-angled, prove that the sum of the two sides containing the right angle is equal to the sum of the diameters of the inscribed and circumscribing circles.

5. Inscribe an equilateral and equiangular hexagon in a given circle.

6. If an angle of a triangle be bisected by a straight line which also cuts the base the segments of the base shall have the same ratio which the other sides of the triangle have to one another.

7. Similar triangles are to one another in the duplicate ratio of their homologous sides.

8. A and B are fixed points, and P is a moveable point such that PA always bears a fixed ratio to PB.

Prove that P lies on a circle, and find the centre of that circle.

9. Define the degree, grade, and unit of circular measure.

The number that denotes an angle when expressed in circular measure is less by 1·5 than that denoting the number of degrees in the same angle. Find the angle.

10. Express all the trigonometrical ratios in terms of the sine.

If the cosecant be twice the sine, find the angle.

11. Prove that

$$\cos(A+B) = \cos A \cos B - \sin A \sin B,$$

stating the limitations assumed in the proof as to the values of A and B.

12. Find $\cos 2A$ and $\sin 3A$.

13. If any arc of a circle be divided into two equal and also into two unequal parts, prove that the rectangle of the chords of the two unequal arcs, together with the square of the chord of the arc between the points of section, is equal to the square of the chord of half the arc.

14. Prove that in any triangle

$$\sin A = \frac{2}{bc}\sqrt{\{s(s-a)(s-b)(s-c)\}}.$$

15. In the triangle ABC

$A = 26^\circ\, 26'$ $b = 127$ and $a = 85$. Find B.

Given $\log 1{\cdot}27 = {\cdot}1038037$,

$\log 8{\cdot}5 = {\cdot}9294189$.

$\log \sin 26^\circ\, 26' = 9{\cdot}6485124$,

$\log \sin 41^\circ\, 41' = 9{\cdot}8228302$,

$\log \sin 41^\circ\, 42' = 9{\cdot}8229721$.

16. If one angle of a triangle be 65°, and one of the sides containing this angle be four times the other, find the remaining angles.

Given $\log 2 = {\cdot}3010300$, $\log 3 = {\cdot}4771213$,

$\log \cot 32^\circ\, 30' = 10{\cdot}1958127$,

$\log \tan 43^\circ\, 17' = 9{\cdot}9739602$,

$\log \tan 43^\circ\, 18' = 9{\cdot}9742133$.

17. The angle of elevation of a tower at a distance of 20 yards from its foot is three times as great as the angle of elevation 100 yards from the same point.

Shew that the height of the tower in feet is $\frac{300}{\sqrt{7}}$.

WOOLWICH.

May, 1873.

ARITHMETIC AND LOGARITHMS.

1. Add together $\frac{1}{6}$ of $\frac{2}{3}$, $\frac{1}{3}$ of $\frac{2}{7}$, and $1\frac{1}{2}$ of $\frac{5}{6}$.

2. Subtract $7\frac{5}{6}$ from $20\frac{3}{10}$.

3. Multiply together $4\frac{2}{3}$, $1\frac{5}{13}$, $\frac{26}{49}$, and $1\frac{7}{72}$.

4. Divide $2\frac{7}{16}$ by $3\frac{1}{8}$.

5. Add together 12·07043, ·7131, 754·5, 6·3, and ·07385.

6. Subtract 82·6874 from 701·212.

7. Multiply ·0263 by 2·36.

8. Divide ·4336218 by 7362.

9. Reduce 16·08 pennyweights to the decimal of a pound troy.

10. Add together $\frac{14}{15}$, $1\frac{4}{25}$, and $5\frac{3}{23}$.

11. Subtract $3\frac{9}{11}$ from $8\frac{2}{3}$.

12. Multiply together $2\frac{2}{7}$ of $1\frac{4}{11}$, $2\frac{5}{8}$ of $\frac{4}{9}$, and $3\frac{1}{2}$ of $\frac{1}{10}$.

13. Divide $4\frac{5}{9}$ by $12\frac{1}{3}$.

14. Add together 6·2 of a day and ·0265 of an hour, and give the answer in minutes and the decimal fraction of a minute.

15. Subtract 6·42 of a furlong from 3·64 of a mile, and give the answer in yards and the decimal fraction of a yard.

16. Multiply ·382 by ·148.

17. Divide 81·27 by ·481 to 3 places of decimals.

18. Reduce ·0325 of £5 to the decimal of £3. 6*s*. 8*d*.

19. Reduce 1 mile 3 furlongs 10 poles 2 feet to feet.

20. If 1 ton 5 cwt. of iron cost £1. 11*s*. 3*d*., what will 1 cwt. 2 qrs. cost?

21. Find (by Practice) the dividend on £8,976. 5*s*. at 13*s*. 3*d*. in the pound.

22. Find the simple interest on £1,760 for 9 years at $3\frac{3}{4}$ per cent. per annum.

23. In 3 acres 2 roods 10 square yards how many square feet?

24. If either 5 oxen or 7 horses will eat up the grass of a field in 87 days, in what time will 4 oxen and 6 horses eat up the same?

25. Find (by Practice) the value of 3 tons and 21 lbs. at £12. 10*s*. per cwt.

26. Find the amount of £6,500 in 3 years at 5 per cent. compound interest (neglecting fractions of a penny).

27. Compute by means of the tables the value of $\frac{(2{\cdot}4806)^8}{(1{\cdot}2701)^{10}}$ to four places of decimals.

28. Given $\log 2 = {\cdot}3010300$, $\log 9 = {\cdot}9542425$, find, *without using the tables*, $\log 5$, $\log 6$, $\log {\cdot}0216$, and $\log \sqrt[5]{({\cdot}375)}$.

29. Extract the square root of 3915380329 and of $83\frac{193}{529}$.

30. Find the present value of £2,587. 18*s*. 9*d*., due 5 years hence, at $3\frac{1}{2}$ per cent.

31. By selling goods for £817. 19*s*. a person lost 9 per cent. on his outlay; find at what price he should have sold them in order to have gained $10\frac{1}{2}$ per cent.

GEOMETRY.

1. Draw a straight line perpendicular to a given straight line of unlimited length from a given point without it.

2. Parallelograms on the same base and between the same parallels are equal to one another.

3. If a straight line be bisected, and produced to any point, the rectangle contained by the whole line thus produced, and the part of it produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

4. If a straight line drawn through the centre of a circle bisect a straight line in it which does not pass through the centre, it shall cut it at right angles.

A straight line is drawn intersecting two concentric circles; prove that the portions of the straight line, intercepted between the two circles, are equal.

5. If a straight line touch a circle, the straight line drawn from the centre to the point of contact shall be perpendicular to the line touching the circle.

6. Describe a circle of given radius passing through a given point and touching a given straight line.

Also describe a circle of given radius touching a given straight line and a given circle.

In each case find how many circles can be drawn.

7. If from a point without a circle two straight lines be drawn, one of which cuts the circle and the other touches it, the rectangle contained by the whole line which cuts the circle and the part of it without the circle shall be equal to the square on the line which touches it.

If two circles intersect each other, their common chord bisects their common tangent.

8. Inscribe in a given circle a triangle equiangular to a given triangle.

9. Triangles and parallelograms of the same altitude are to one another as their bases.

10. Find a mean proportional between two given straight lines.

11. Similar triangles are one to another in the duplicate ratio of their homologous sides.

ALGEBRA.

1. If $x = 4$, $y = 3$, find the numerical value of

$$(x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4),$$

and show that for these values of x and y, $\sqrt[4]{(x^3 + x^2 + 1)} = y$.

2. Reduce to their simplest forms the following expressions

$$(1)\quad \frac{x^2 - xy + y^2}{x - y} - \frac{x^2 + xy + y^2}{x + y}.$$

$$(2)\quad \{a + b\sqrt{(-1)}\}^2 \times \{a - b\sqrt{(-1)}\}^2.$$

3. Investigate a rule for finding what is called the least common multiple of two algebraical expressions. Find the least common multiple of $(x - a)$ and $x^{\frac{2}{3}} - 2x^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}$.

Reduce to its lowest terms $\dfrac{acx^2 - (ad + bc)\,x + bd}{a^2x^2 - b^2.}$.

4. Solve the following equations:

$$(1)\quad (x-1)^3 + (x-2)^3 + (x-3)^3 = 3(x-1)(x-2)(x-3).$$

$$(2)\quad \left.\begin{aligned} &\frac{12}{x} + \frac{12}{y} = 7, \\ &\frac{8}{x} - \frac{9}{y} + \frac{11}{12} = \frac{1}{x} - \frac{1}{y}. \end{aligned}\right\}$$

$$(3)\quad (2x - 1)(3x + 1) - (x - 1)(2x + 1) = 26.$$

$$(4)\quad \left.\begin{aligned} &x^2 + y^2 + z^2 = 77, \\ &xy + yz - xz = 26, \\ &y + z = 11. \end{aligned}\right\}$$

5. A person bought two farms, one of pasture, the other of arable land, the latter exceeding the former by 20 acres; for the pasture land he paid £10 an acre more than he paid for the arable, and the whole cost of the pasture farm was £9,000 and of the arable £8,000. How many acres were there in each farm, and what price per acre was paid in each case?

6. Express algebraically the condition that one quantity shall vary directly as a second and inversely as a third. State any known geometrical example of such a law of variation.

Given that y is equal to the sum of two terms, one of which varies as $\frac{1}{x}$ and the other as x^2, and that when x is 1 or 2, y is equal to 7, find the equation between y and x.

7. A merchant transmits to a retailer a cask of spirits containing (a) gallons, the cask is consigned to three carriers in succession, the first when he receives it draws off one-fourth of a gallon from the cask and fills it up with water, the second when he receives it does likewise, and so does the third, find the ratio of the quantity of spirit in the cask to the quantity of water when the retailer receives it, and find the actual quantities of water and spirit remaining if (a) is nine gallons.

8. Without assuming the general expression for the sum of an arithmetic series, find the sum of (n) terms of the series

$$\frac{a}{n^2}, \frac{2a}{n^2}, \frac{3a}{n^2}, \text{\&c.}$$

There are (x) terms in arithmetical progression between 5 and 25, and the second term is $\frac{3}{7}$ths of the last term but one, express the terms of the series.

9. Find a geometrical mean between

$$\left(1-\frac{2x}{3}+\frac{x^2}{9}\right) \text{ and } \left(1-\frac{4x}{3}+\frac{4x^2}{9}\right).$$

If a, b, c are in harmonical progression, prove $a^3+c^3>2b^3$.

10. Distinguish between ' permutations and combinations. Deduce from first principles the expression for the number of permutations of (n) things taken (4) at a time, and assuming the general form, show that the number of permutations of (n) things taken altogether when they all differ is double the number of permutations of (n) things taken together when two of them are alike.

11. Assuming the form of the expansion of a binomial, express the $(r+1)^{\text{th}}$ term of $(1+x)^{-n}$; show how to find the greatest term of $(1+x)^{-n}$ when (x) is a proper fraction and (n) an integer, determine the place of the greatest term in the expansion of $(1+\frac{4}{7})^{-4}$.

Pure Mathematics.

1. Prove that

$$(y+z-x)^2+(z+x-y)^2+(x+y-z)^2=4(x^2+y^2+z^2)-(x+y+z)^2,$$

and that $(a^2+b^2)(c^2+d^2)=(ac\pm bd)^2+(ad\mp bc)^2$.

In how many ways can the expression

$$(a^2+b^2)(c^2+d^2)(e^2+f^2)$$

be written in the form of the sum of two squares?

2. Solve the equation

$$6x^2-89x+323=0.$$

If y and z be the roots of the equation $ax^2+bx+c=0$, prove that $a(y+z)=-b$, and $ayz=c$.

Find, in terms of a, b, c, the values of the expressions y^2+z^2, and y^3+z^3.

3. Sum the series

$$1 + 3 + 5 + 7 + \ldots \text{ to } n \text{ terms},$$

$$1 + 3 + 9 + 27 + \ldots \text{to } n \text{ terms},$$

and $$1^2 + 2^2 + 3^2 + 4^2 + \ldots \text{ to } n \text{ terms}.$$

4. Write down the coefficients of x^3, and of x^r, in the expansions of $(1-x)^{-2}$, and $(1-x)^{-\frac{1}{2}}$.

If $$\frac{n(n-1)(n-2)\ldots(n-r+1)}{1.2.3\ldots r} = {}_nC_r,$$

prove that $${}_{n+1}C_r = {}_nC_r + {}_nC_{r-1}.$$

5. Express $\frac{200}{119}$, and $\sqrt{(10)}$ in the form of continued fractions, and find the first four convergents to each of the continued fractions.

6. Define the sine, and the cosine, of an angle; and prove the formulæ

$$\sin A = \sin(180^\circ - A),$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$

Find the values of $\sin 15^\circ$ and $\sin 75^\circ$.

7. Prove that in any triangle ABC, of which a, b, c are the sides,

$$b^2 + c^2 - a^2 = 2bc \cos A.$$

Find the angles of a triangle, the sides of which are 6 inches, 3 inches, and 3 $\sqrt{(3)}$ inches.

8. If R be the radius of the circumscribing circle of a triangle ABC, prove that

$$2R = \frac{a}{\sin A}.$$

State the *ambiguous case* in the solution of triangles, and prove that, in that case, the circumscribing circles of the two triangles are equal.

9. Define the logarithm of a number to a given base, and find the logarithms of 64 to the base 2, and of 128 to the base $\frac{1}{2}$.

Prove that $\log(m \times n) = \log m + \log n$.

10. Explain how probability is measured. A bag contains five sovereigns and ten shillings; if four coins be taken out at random, find the chance that two sovereigns and two shillings will be taken out.

11. Prove that an equation of an odd degree has at least one real root; and that an equation of an even degree with its last term negative has at least two real roots of contrary signs.

Can the equation

$$x^6 + 3x^4 + 4x^2 + 5 = 0$$

have any real root?

12. Find the equation to a straight line referred to rectangular axes.

Determine the equation to the straight line which passes through the point (1, 2) and is perpendicular to the line $y = x + 1$.

13. Define a parabola, and find its equation in the form $y^2 = 4ax$.

Show that the equation to the tangent at the point (x', y') is

$$y - y' = \frac{2a}{y'}(x - x');$$

and that if p be the length of the perpendicular upon it from the vertex

$$p^2(a + x') = ax'^2.$$

14. Prove that the equations

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ and } \frac{x^2}{a^2 + c^2} + \frac{y^2}{b^2 + c^2} = 1$$

represent ellipses having the same foci.

Show that the ellipse $x^2 + 4y^2 = 4$, and the hyperbola $x^2 - 2y^2 = 2$, have the same foci; find the coordinates of their point of intersection, and draw figures of the two curves.

MECHANICS AND HYDROSTATICS.

1. Define the terms *resultant force* and *moment of a force about a point.*

Assuming that the moments of two intersecting forces about every point in the line of action of their resultant are equal and opposite, prove the proposition of the parallelogram of forces both as to direction and magnitude.

2. If two equal particles be situated at the extremities of the diameter of a circle, each attracting with a force varying as the distance, prove that the resultant force on any particle on the circumference of the circle is the same for all positions of the particle, and always passes through the centre of the circle.

3. Prove that whenever two parallel forces can be replaced by a single resultant, the moments of the forces about any point in the line of action of the resultant are equal and opposite.

In what case is there no single resultant, and what is the system called in this case?

4. ACB is a bent lever, fulcrum C, and arms inclined at an angle of 135°. When a weight of 2 lbs. is suspended at A, a force of 1 lb. acting perpendicularly to the arm CB at B will keep AC horizontal, and if AC be lengthened and CB shortened, each by one foot, two weights of 2 lb. and 1 lb. respectively suspended at A and B will keep BC horizontal; find the lengths of AC and CB.

5. Enunciate the principle of the centre of parallel forces, and apply it to find the centre of gravity of a triangle and of the curved surface of a cone.

If two triangles be formed by joining the middle points of alternate sides of any hexagon, prove that their centres of gravity are coincident.

6. State the laws of statical friction.

A particle of given weight is placed on a rough inclined plane, and attached to a string which passes over the highest horizontal line in the plane, and hangs vertically over it. Find the greatest and least weights which can be attached to the free end of the string consistently with equilibrium.

7. Define acceleration; and assuming that the acceleration of gravity is 32 when a foot and a second are the units of space and time respectively, find its numerical value when a yard and a minute are taken as the units.

8. Explain the proposition of the parallelogram of velocities.

If a railway train be moving at the rate of 20 miles an hour, and a heavy body fall from the roof to the floor of a carriage in one second, find the inclination of the path of the body to the horizon in fixed space at the instant before the end of its fall, the acceleration of gravity being 32 on the usual assumption as to units.

9. The path of a projectile in vacuo is a parabola.

A bombshell on striking the ground bursts, scattering its fragments with a mean velocity v; find the area of ground covered by the fragments, assuming that it falls on a level surface. If it falls on a road running up a hill, find the greatest distances reached up and down the road respectively.

10. Find the conditions of equilibrium of a body floating in a fluid.

11. Define, and give equations for finding, the centre of pressure of a plane surface in contact with a heavy fluid at rest.

A cubical box has a heavy lid moveable about hinges on one of the edges. If the box be filled with water and placed on a rough inclined plane with the line of hinges horizontal, find the inclination of the plane that the water may just begin to flow out.

The line of hinges being towards the upper end of the plane.

12. State the relation between the pressure, density, and temperature of a gas.

Explain the construction and action of the barometer.

If a tube of given length, and closed at one end, be filled with atmospheric air, and the open end be immersed to a given depth below the surface of mercury in a cistern, find how high the mercury will rise in the tube, the height of the mercurial barometer being h, and the temperature of the air in the tube being constant.

Any of the following questions may be substituted for an equal number of the above.

A. Define, and find the equations of the central axis of a given system of forces on a rigid body.

Ex. Three given forces acting along three edges of a cube, no two intersecting and no two parallel.

B. If the attached weight in No. 6 be intermediate between the greatest and least weights consistent with equilibrium, find the inclination of the string to a horizontal line in the plane when there is equilibrium; also find the direction in which the weight on the plane will begin to move if the attached weight be increased.

C. The resultant attraction of a uniform straight line upon any particle bisects the angle subtended by the line at the particle.

D. Prove the equation $h^2u^2\left(\frac{d^2u}{d\theta^2}+u\right)=P$ in central forces.

PURE MATHEMATICS.

1. In any spherical triangle prove that the sines of the angles are proportional to the sines of the opposite sides.

2. Enunciate Napier's rules for the solution of right-angled spherical triangles, and prove them for the cases in which one of the sides containing the right angle is the middle part.

3. If ε be the spherical excess of any spherical triangle, prove that

$$\cot\frac{\varepsilon}{2} = \frac{\cot\frac{a}{2}\cot\frac{b}{2} + \cos C}{\sin C}.$$

4. Find from definition the differential coefficients of x^n (n unrestricted) and of $\sin x$.

Differentiate

$$(1)\ \left(\frac{a+x}{a-x}\right)^{\frac{3}{2}},\quad (2)\ (\log x)^x,\quad (3)\ \cos(\tan^{-1}x).$$

5. Find the r^{th} differential coefficient of $x^n e^{mx}$, and prove that if

$$y^n - ny + (n-1)\,x = a,\ \text{then}\ \frac{d^2y}{dx^2} - ny^{n-2}\left(\frac{dy}{dx}\right)^3 = 0.$$

6. Write down Taylor's expansion for $f(x)$, with the remainder after n terms.

Expand e^{ex} to 4 terms, and find the first four terms of the expansion of y in terms of x in the equation

$$x^2 + y^2 - 6x + 4y + 4 = 0.$$

7. State fully the conditions that $f(x)$ may be a maximum or a minimum.

Apply to $3x^4 + 8x^3 + 6x^2 + 3$.

Upon the same circular base, and upon opposite sides of it, two right cones are constructed so that the volume of the whole figure thus formed may have a given value. Find the condition that the surface of the figure may be a minimum.

8. Find the equation of the tangent to the curve $y = f(x)$.

Apply to the curve

$$y = \frac{a}{2}\left(\varepsilon^{\frac{x}{a}} + \varepsilon^{-\frac{x}{a}}\right).$$

9. Find any expression for the radius of curvature at any point of a curve, and apply it to the curve in the last question.

10. Prove the formula $\frac{dA}{d\theta} = \frac{r^2}{2}$ where A is the sectorial area subtended by a curve at the origin.

If r be the distance of any point of an ellipse from one of the foci, and a be the semi-major axis, prove that $\int \sqrt{\left(\frac{2a - r}{r}\right)}\, ds$ along any arc is proportional to the sectorial area subtended by that arc at the other focus.

11. Integrate the expressions

$$(1)\ \sqrt{(a^2 - x^2)}, \qquad (2)\ \frac{1}{1 + \tan x};$$

and find a formula of reduction for the integration of $(a^2 + x^2)^{\frac{n}{2}}$.

12. Find the equation of a plane referred to three rectangular axes.

The planes $2x + 3y + 4z = a$ and $5x + 4y + 3z = b$ intersect the plane, $x = c$.

Find the cosine of the angle between the lines of intersection.

13. Interpret the equations

$$(1)\ (x - a)^2 + (y - b)^2 + (z - c)^2 = k^2\{(x - a)\, l + (y - b)\, m + (z - c)\, n\}^2.$$

$$(2)\ (x - a)^2 + (y - b)^2 + (z - c)^2 = \{xl + ym + zn\}^2,$$

where $l^2 + m^2 + n^2 = 1$ in each case.

Prove that the surfaces thus represented intersect in plane curves, and find the equations to the planes of such intersection.

Any of the following questions may be substituted for an equal number of the above.

A. If the curve in question 8 have contact with a circle of radius r, whose centre is in the axis of y, and if the angle of inclination of the common tangent to the axis of x be 45°, prove that

$$a = \frac{r}{\sqrt{(2)} \log_e \{1 + \sqrt{(2)}\}}.$$

Determine also whether the radius of curvature at the point of contact is greater or less than that of the circle.

B. How are solid angles measured?

Prove that the solid angle subtended by a square whose side is $2a$ at a point in the perpendicular to its plane through its centre and at a distance b from its plane is to the solid angle subtended by any closed surface surrounding the same point as $\sin^{-1} \dfrac{a^2}{a^2 + b^2}$ is to π.

C. Find the asymptotes and any singular points of the curves

$$(1)\ y^2 = \frac{ax^2 (x - a)}{x^2 - 4a^2}, \quad (2)\ r = a \frac{(3\theta - \pi)(5\theta - \pi)}{4\theta - \pi},$$

and trace the curves.

WOOLWICH.

September, 1873.

ARITHMETIC AND LOGARITHMS.

1. Add together $\frac{3}{4}$ of $\frac{2}{9}$, $\frac{7}{12}$ of $\frac{3}{10}$, and $5\frac{1}{2}$ of $\frac{9}{11}$.

2. Subtract $4\frac{19}{36}$ from $6\frac{1}{12}$.

3. Multiply together $5\frac{6}{7}$, $\frac{27}{32}$, $2\frac{25}{82}$, and $\frac{8}{9}$.

4. Divide $5\frac{12}{17}$ by $\frac{13}{51}$.

5. Add together ·7131, 754·5, 6·3, ·07385, and 891·062.

6. Subtract 378·0695 from 400·021.

7. Multiply 57·01 by ·00284.

8. Divide 80·14662 by 236·7.

9. Reduce 12*s.* $11\frac{1}{4}$*d.* to the decimal of £2. 11*s.* 9*d.*

10. Add together $7\frac{1}{5}$, $\frac{4}{35}$, and $1\frac{3}{14}$.

11. Subtract $10\frac{4}{11}$ from $12\frac{3}{22}$.

12. Multiply together $7\frac{2}{5}$ of $3\frac{1}{8}$, $5\frac{1}{4}$ of $\frac{3}{7}$, and $\frac{2}{5}$.

13. Divide $8\frac{7}{8}$ by $15\frac{3}{4}$.

14. Add together 15·63 of a day and 4·01625 of an hour, and give the answer in minutes and the decimal fraction of a minute.

15. Subtract 5·72 of a pole from ·256 of a mile, and give the answer in yards and the decimal fraction of a yard.

16. Multiply 314·5 by 6·28.

17. Divide 410·9 by 3·26 to 3 places of decimals.

18. Reduce 5·056 dwts. to the decimal of a pound troy.

19. Reduce 5 m. 2 fur. 25 p. 4 yds. to inches.

20. If 7 cwt. 1 qr. of sugar cost £26. 10*s.* 4*d.*, what will be the price of 2 tons 3 cwt. 2 qrs.?

21. Find (by Practice) the dividend on £6,052. 13*s.* 4*d.* at 17*s.* $9\frac{3}{4}$*d.* in the pound.

22. Find the simple interest on £2,350 for 6 years at $7\frac{2}{3}$ per cent. per annum.

23. In 5 acres 3 roods 25 square yards how many square feet?

24. If 5 acres contain grazing for 20 sheep for 7 days, how many will be required for 52 sheep for 35 days?

25. Find (by Practice) the value of 79 tons 8 cwt. 3 qrs. at £18. 3*s.* 4*d.* per ton.

26. Find the compound interest on £7,850 in 3 years at 4 per cent. (neglecting fractions of a penny).

27. Compute by means of the tables the value of $\frac{(3\cdot05)^6}{61\cdot28} \times \sqrt[7]{(3056\cdot25)}$ to five places of decimals.

28. Given

$$\log 2 = \cdot3010300, \ \log 3 = \cdot4771213, \ \log 13 = 1\cdot1139434;$$

find $\log \sqrt[3]{\frac{24}{135}}$, $\log \cdot01625$, and find how many figures the number represented by 2^{32} is composed of.

29. Extract the square root of 255200625 and of $37\frac{212}{841}$.

30. How much money must be invested in the 3 per cent. consols at $94\frac{1}{4}$, so as to yield an income of £225 per annum?

EUCLID.

1. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are together equal to two right angles.

Prove that the angle between the bisectors of two consecutive angles of a quadrilateral is half the sum of the two remaining angles.

2. Parallelograms on equal bases, and between the same parallels, are equal to one another.

3. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

If on the larger of the two parts into which the given straight line is divided, a portion be marked off equal to the smaller of these parts, prove that the larger part will be divided in the same way as the given straight line.

4. If two circles touch one another internally, the straight line which joins their centres, being produced, shall pass through the point of contact.

5. The angles in the same segment of a circle are equal to one another.

Of all the triangles upon the same base, and having the same vertical angle, prove that the isosceles triangle has the greatest area.

6. If AD, BE, and CF be perpendiculars from A, B, and C respectively, upon the opposite sides of the triangle ABC, prove that AD bisects the angle EDF.

7. If from any point without a circle two straight lines be drawn, one of which cuts the circle, and the other touches it; the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, shall be equal to the square on the line which touches it.

8. Inscribe a circle in a given triangle.

If the triangle be right-angled, prove that the two sides containing the right angle are together equal to the sum of the diameters of the inscribed and circumscribing circles.

9. If the exterior angle of a triangle, made by producing one of its sides, be bisected by a straight line which also cuts the base produced, the segments between the dividing straight line and the extremities of the base shall have the same ratio which the other sides of the triangle have to one another.

10. Divide a given arc of a circle into two parts whose chords shall be in a given ratio.

11. If from the vertical angle of a triangle a straight line be drawn perpendicular to the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained by the perpendicular and the diameter of the circle described about the triangle.

Prove that the ratio of the diameter of the circle circumscribing a triangle to any one of the sides is equal to the ratio of half the rectangle of the two remaining sides to the area of the triangle.

ALGEBRA.

1. Find the values of $(x-10)^2+(x-6)^2$ when $x=5$, and when $x=8+2\sqrt{(-1)}$. Explain why any real numerical value substituted for x in the above expression will always give a positive result. If $5^x=\frac{1}{625}$, find x.

2. Reduce to their simplest forms

(1) $(x+a+b)(x-a-b)(x+a-b)(x-a+b)$,

$$\frac{(x^4-4x^3y+6x^2y^2-4xy^3+y^4)\times(x^2+2xy+y^2)}{(x^2-y^2)^2}.$$

3. Prove

$$(1)\quad \frac{2n(2n-1)(2n-2)}{1.2.3}-\frac{(2n)^2(2n-1)}{1.2}+\frac{(2n)^2(2n+1)}{1.2}-\frac{2n(2n+1)(2n+2)}{1.2.3}=0.$$

$$(2)\quad \frac{x^{\frac{3}{2}}+y^{\frac{3}{2}}+xy(x^{-\frac{1}{2}}+y^{-\frac{1}{2}})}{x^{\frac{3}{2}}-y^{\frac{3}{2}}-xy(x^{-\frac{1}{2}}-y^{-\frac{1}{2}})}=\frac{x+y}{x-y}.$$

4. If $\sqrt{\{a+\sqrt{(b)}\}}=x+y$, where (a) is a rational quantity, $\sqrt{(b)}$ a quadratic surd, x and y one or both quadratic surds; prove $\sqrt{\{a-\sqrt{(b)}\}}=x-y$. Extract the square root of $7-2\sqrt{(10)}$. Reduce

$$(3^{\frac{5}{3}}-2^{\frac{1}{2}}3^{\frac{4}{3}}+6-2^{\frac{3}{2}}3^{\frac{2}{3}}+2^2 3^{\frac{1}{3}}-2^{\frac{5}{2}})\times(3^{\frac{1}{3}}+2^{\frac{1}{2}}).$$

5. Solve the following equations:

$$(1)\quad \left.\begin{aligned} \frac{7x+5y-12}{7x-5y+12} &= \frac{29}{6} \\ \frac{7x}{5} - \frac{5y}{7} &= \frac{4x+8y-6}{35} \end{aligned}\right\}.$$

$$(2)\quad \frac{x+5}{x-3} - \frac{x-3}{x+5} = \frac{80}{9}.$$

$$(3)\quad (x-3)^3 + (x-4)^3 = 43\{(x-3)^2 - (x-4)^2\}.$$

6. A clothier sold an army tailor 60 yards of scarlet cloth and 120 yards of coarse blue cloth for £49. 10*s.*, and for £2. 5*s.* he sold 4 yards more of the blue cloth than of the scarlet cloth; what was the price per yard of each cloth?

7. Eliminate x between the equations

$$x^3 - px^2 + qx - r = 0 \text{ and } x^2 - y = 0;$$

express the resulting equation free from surd quantities and arranged according to decreasing integral powers of y.

8. Write down n^{th} term of the arithmetical series of which the two first terms are $a-3b$, $a-b$, show that the sum of every pair of terms equidistant from the beginning and end of the series is invariable, and express the sum of (n) terms of the series.

The first term of an arithmetic series is 8, the common difference 4, and the sum of the series 108, find the number of terms and show how the negative value of (n) obtained from the formula is to be interpreted.

9. The condition that three quantities shall be in harmonical progression can be expressed by a property of proportion; state that property, and show that it is fulfilled by the three quantities $\frac{c}{a+b}$, $\frac{c}{a+2b}$, $\frac{c}{a+3b}$.

The arithmetic mean between A and B is $(1+x^2)^2$ and the harmonic mean $(1-x^2)^2$, find the geometric mean; determine also A and B.

10. Assuming the general formula for the number of combinations of (n) things taken (r) at a time show that the formula will give the same number whether $r=4$ or $r=(n-4)$, verify this when n is 10; also from the theory of combinations show that the product of any (n) consecutive integers is divisible by $(1.2.3...n)$.

11. Assuming the form of the binomial theorem when the index is integral and positive, prove the theorem when the index is integral and negative. When the index (n) is positive, show that the r^{th} coefficient is greater than the $(r+1)^{\text{th}}$ coefficient if r be greater than $\frac{n+1}{2}$, and when (n) is integral and negative, show that the coefficients continually increase.

Find by the binomial theorem an approximate value to $(1013)^{\frac{1}{3}}$.

PURE MATHEMATICS.

1. Express $\sqrt{(11)}$ and $4-\sqrt{(11)}$ in the form of a continued fraction.

Find the value of

$$\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{2}}} \text{ \&c. ad infinitum.}$$

2. Find a solution in positive integers of the equation $7x+11y=245$.

Find also the number of solutions admissible.

3. Given that

$$a^x = 1 + a_1 x + a_2 x^2 + a_3 x^3 + \text{\&c. ad infinitum.}$$

Find a_2, a_3, &c. in terms of a_1.

4. Prove that $\log a^m = m \log a$ to any base.

Given that $\log_{10} 1{\cdot}2589 = {\cdot}0999912$,

$\log_{10} 1{\cdot}2590 = {\cdot}1000257.$

Find $^{10}\sqrt{(10)}$ to 5 places of decimals.

5. Find the chance of throwing 6 with four counters, each of which is marked with 1 on one side and 2 on the other.

6. Prove that $\cos(A + B) = \cos A \cos B - \sin A \sin B$. Find also $\sin 3A$.

7. Prove that in any triangle ABC whose sides are a, b, and c,

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

Given $b = 2$, $c = 3$, $\cos A = \frac{1}{3}$. Find a, $\cos B$, and $\cos C$.

8. If p be the greatest negative coefficient of an equation, prove that $p + 1$ is a superior limit of the roots.

9. *State* how the derived function $f'(x)$ may be employed to ascertain the existence of equal roots in the equation $f(x) = 0$.

Ex. $$x^3 - 5x^2 - 8x + 48.$$

10. Find the equation of a straight line referred to rectangular coordinates.

Find the equation of the line joining the origin to the point of intersection of the lines

$$2y - x + a = 0,\quad 2x + y - a = 0.$$

11. Write down the equation of the tangent to the parabola $y^2 = 4ax$ at the point x', y'.

If this tangent intersect a second parabola having the same axis and vertex as the first but with a greater latus rectum, prove that the ordinate of the point of contact of the tangent with the first parabola is a harmonic mean between the ordinates of the points of intersection with the second parabola.

12. If P and D be the extremities of a pair of conjugate diameters of an ellipse, and ϕ and ϕ' the eccentric angles at those points, prove that $\phi' = \frac{\pi}{2} + \phi$.

Any of the following may be substituted for an equal number of the above.

A. If p be a prime number and a be prime to p, prove that

$$a^{p-1} - 1$$

is divisible by p.

B. Prove De Moivre's theorem and apply it to solve the equation

$$x^8 - 1 = 0.$$

C. If ABC be a triangle, and D, E, and F be the centres of the escribed circles, prove that the area of the triangle DEF is

$$8R^2 \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2},$$

R being the radius of the circumscribing circle.

D. Prove that the equation,

$$Ax^2 + Bxy + Cy^2 = 1,$$

represents an ellipse or hyperbola according as $B^2 <$ or $> 4AC$.

Find the eccentricity.

MIXED MATHEMATICS.

1. Assuming the parallelogram of forces for direction, prove it also for magnitude.

A picture is supported by a string passing through two smooth rings in the frame and over a smooth nail in the wall. If the string be lengthened will its tension be increased or diminished?

2. Find the magnitude and line of action of the resultant of two parallel forces acting on a rigid body in the same direction.

3. Find the C.G. of a uniform plane triangle.

If such a triangle be supported by three vertical strings attached to the angular points, prove that the tensions of the strings will be equal. If the point of attachment of one of the strings be shifted half way towards the C.G., how will the tensions be affected?

4. State the laws of friction.

A heavy body rests on a rough horizontal plane and is pulled by a string.

Find the angle of inclination of the string to the plane when the tension which just moves the body is the least possible, and the magnitude of the tension in this case.

5. Prove the formulæ:

$$s = \frac{ft^2}{2}, \quad v^2 = 2fs,$$

in uniformly accelerated motion.

6. Prove that the path of a projectile in vacuo is a parabola.

A body is projected horizontally from the top of a tower of given altitude. Find the velocity of projection in order that it may strike the ground at an angle of 45°.

7. Two unequal heavy bodies are connected by a string passing over a smooth pulley.

Find the acceleration and tension of the string.

Supposing that one of the bodies is twice as heavy as the other, that they are in the same horizontal plane after moving for three seconds, and that the string breaks at that instant, find their distance apart at the end of the next second.

8. Prove that the pressure at any point in a heavy uniform fluid at rest varies as the depth below the surface.

9. Find the conditions of equilibrium of a body floating in a fluid.

10. A cubical vessel rests on a horizontal base, and is half filled with water.

A solid cube, whose edge is half that of the vessel, and specific gravity half that of water, is thrown in.

Find by what part of themselves the pressures on the sides and base of the vessel are respectively increased.

11. State the relation between the pressure, density, and temperature of a gas.

Describe the construction and action of Smeaton's air pump.

How is this action affected by raising the temperature of the receiver above that of the external air?

Any of the following questions may be substituted for any of the above.

A. Find the condition that a system of forces acting on a rigid body should admit of a single resultant.

Ex. Three forces acting along the edges of a cube, no two parallel, and no two intersecting.

B. In motion about a centre of force, prove that the velocity at any point of the orbit is that due to $\frac{1}{4}$th of the chord of curvature through the centre of force.

C. Prove the equation

$$dp = \rho\ (Xdx + Ydy + Zdz),$$

where ρ is the density, p the pressure, and X, Y, Z, the component parts of the force at any point of a fluid at rest.

If the fluid be homogeneous and incompressible, find the condition which must be satisfied by X, Y, Z, in order that equilibrium may be possible.

PURE MATHEMATICS.

1. In spherical trigonometry what are the poles of great circles? Given a great circle show how to find its poles. Prove that the angular points of a polar triangle are the poles of the sides of its primitive triangle.

2. Given the three angles of a spherical triangle find the cosine of one of the sides. If the triangle be equilateral express the cosine of a side in the form adapted to logarithmic computation.

Find a side of the equilateral triangle whose angles are each 120°.

3. If a, b, c be the sides, subtending the angles A, B, C, of a spherical triangle,

prove $$\tan\frac{a+b}{2} = \frac{\cos\frac{A-B}{2}}{\cos\frac{A+B}{2}}.\tan\frac{c}{2}.$$

If $a=b$, show that the formula expresses a result that can be obtained by Napier's rules for the solution of right-angled triangles.

4. If $y=f(x)$ be a function of one independent variable and x receive an increment (h), distinguish between the *difference* of y and the *differential* of y with respect to the increment of (x).

Illustrate this when $y=ax^3$.

If $y=\sin x$ show how to obtain $\frac{dy}{dx}$.

5. Differentiate the following functions:

(1) $y^2=\frac{b^2}{a^2}(a^2-x^2)$.

(2) $y=\log_e\{x+a+\sqrt{(2ax+x^2)}\}$.

(3) $y=\tan^{-1}\frac{x}{\sqrt{(1-x^2)}}$.

(4) If $y=x^3\log_e x$, find $\frac{d^4y}{dx^4}$.

6. If $u = f(x)$ can be expanded into a series of the form $u = A + Bx + Cx^2 + Dx^3 + \&c.$, show how to find the coefficients, and express the general form of the coefficient of x^n.

Assuming that $\log_e(1+x)$ can be so expanded, find the series.

Expand to three terms $\left(\frac{1+e^{-x}}{2}\right)^n$.

7. If $u = \frac{P}{Q}$, where P and Q are functions of x which separately vanish for a particular value of x, show generally how (u) may be determined by the differentiation of P and Q. Give any example when this method will fail.

Find (u) in the following examples:

(1) $u = \frac{x^3 - a^3}{x^3 + 2ax^2 - a^2x - 2a^3}$ when $x = a$.

(2) $u = \frac{2}{1-x^2} - \frac{3}{1-x^3}$ when $x = 1$.

8. Find the cylinder of greatest convex surface that can be inscribed in a given hemisphere, the centres of the base of the cylinder and of the hemisphere coinciding, and show by the differential test that the surface is a maximum.

9. Prove that the perpendicular from the origin on the tangent at a point xy of a plane curve referred to rectangular coordinates is

$$\frac{y - x\frac{dy}{dx}}{\sqrt{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}}}.$$

Apply this expression to prove that in the common parabola the perpendicular from the focus on the tangent is a mean proportional between the focal distance and the distance of the focus from the vertex.

10. Find the differential expression for the subtangent of a curve referred to polar coordinates.

If S be the pole, ST the sub-tangent, SY the perpendicular from S on the tangent at P, prove

$$\frac{1}{SY^2} = \frac{1}{SP^2} + \frac{1}{ST^2}.$$

11. Obtain the differential expression for finding the content of a solid of revolution, and compare the content of a paraboloid measured from the vertex with that of the circumscribing cylinder.

12. Integrate

$$(1)\ \int \frac{dx}{(x-3)(x-5)} \quad (2)\ \int \frac{x^2 dx}{\sqrt{(a^2-x^2)}} \quad (3)\ \int \frac{dx}{\sqrt{(2ax-x^2)}}.$$

13. In reference to coordinate geometry of three dimensions, explain why a plane is defined by one equation and a straight line by two equations.

If $Ax + By + Cz = D$ be the equation to a plane referred to rectangular axes, determine the geometrical significance of the constants in the equation; and find also the cosine of the angle which the plane makes with the coordinate plane of xy.

EXAMINATION OF CANDIDATES FOR COMMISSIONS IN THE ROYAL MARINE ARTILLERY AND LIGHT INFANTRY.

August, 1873.

MATHEMATICS, SECTION I.

ARITHMETIC AND LOGARITHMS. 3 hrs.

1. Add together $\frac{9}{10}$, $\frac{11}{12}$, and $\frac{7}{8}$.
2. From $1\frac{5}{13}$ subtract $\frac{6}{19}$.
3. Multiply $8\frac{9}{11}$ by $9\frac{3}{8}$.
4. Divide $8\frac{5}{6}$ by $4\frac{1}{4}$.
5. Add together 3·07, ·006, 14·13, and ·00179.
6. From 3·062 subtract 1·0043.
7. Multiply 8·64 by $\frac{3}{25}$.
8. Multiply 3·02 by 1·0201.
9. Divide ·0304 by 64.
10. Add together $3\frac{1}{5}$, $10\frac{8}{9}$, $\frac{19}{20}$, and $1\frac{1}{4}$.
11. From $\frac{2}{15}$ subtract $\frac{1}{2}$ of $\frac{1}{7}$.
12. Multiply $\frac{2}{7}$ of $5\frac{1}{2}$ by $\frac{4}{5}$ of $\frac{6}{7}$.
13. Divide $3\frac{5}{8}$ by $3\frac{2}{3}$.
14. Express £3. 2*s*. 6*d*. as a fraction of 8 guineas.
15. In 2 acres 1 rood 4 perches how many square yards?
16. Divide 1 by 236 to 5 places of decimals.
17. Convert ·965 into a vulgar fraction in the lowest terms.

18. Express $\frac{1}{14}$ as a recurring decimal.

19. Find the number of seconds in ·869 of a day.

20. Find the value of ·565 of a cwt., at 1*s.* 3*d.* the pound.

21. Divide ·6607 by 228.

22. Find the simple interest on £325 for 8 months at 6 per cent. per annum.

23. If a commission of three pence in the pound on a sum of money is £3. 16*s.*, what is the sum of money?

24. If 2 qrs. 13 lbs. cost £10. 18*s.* 6*d.*, what will a ton cost?

25. Find the square root of 59136100.

26. Employ logarithms to compute $(\cdot 030672)^{\frac{1}{3}}$ to six places of decimals.

27. Find by logarithms the compound interest in 7 years on £326. 15*s.*, at 2½ per cent. per annum.

28. By logarithms divide 1237 by ·00864.

29. What sum is reduced by a discount of 3 per cent. to £36. 7*s.* 6*d.*?

MATHEMATICS, SECTION I. EUCLID. 3 hrs.

1. If two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise their bases equal, the angle which is contained by the two sides of one of them shall be equal to the angle contained by the two sides equal to them of the other.

2. Define a parallelogram.

The complements of the parallelograms which are about the diameter of any parallelogram are equal to one another.

In the case of a square, if the squares about the diameter are 169 and 625 square feet in area, what is the area of each complement?

3. If a straight line be bisected and produced to any point, the square on the whole line thus produced and the square on the part of it produced, are together double of the square on half the line bisected, and the square on the line made up of the half and the part produced.

4. Of all straight lines in a circle drawn through a given point within it, the longest is the diameter of the circle, and the shortest is the straight line perpendicular to this diameter.

5. When are segments of circles similar?

Upon the same straight line and upon the same side of it there cannot be two similar segments of circles not coinciding.

6. If from a point without a circle there be drawn two straight lines, one of which cuts the circle and the other meets it, if the rectangle contained by the whole line which cuts the circle and the part of it without the circle be equal to the square on the line which meets it, the line which meets shall touch the circle.

7. When is a straight line said to be placed in a circle?

In a given circle to place a straight line equal to a given straight line not greater than the diameter of the circle.

8. To inscribe a square in a given circle.

9. If a straight line be drawn to cut two sides of a triangle proportionally, this line shall be parallel to the remaining side of the triangle.

10. Define duplicate ratio.

Similar triangles are to one another in the duplicate ratio of their homologous sides.

11. Define a plane. State when a straight line is perpendicular to a plane, and when two planes are perpendicular to one another.

Planes to which the same straight line is perpendicular are parallel to one another.

MATHEMATICS, SECTION I.—ALGEBRA. 3 hrs.

1. If $x = 4$ and $y = -3$, find the value of

$$\frac{\sqrt{(x^2+y^2)}}{x+y} + 3\,\frac{x^3-y^3}{\sqrt{(x+y)}}.$$

2. Subtract

$(x^2-5)^2$ from $(x-3)(x-1)(x+1)(x+3)$.

3. Divide

$(a+b)^2 - 3(a+b)\sqrt{(ab)} + 2ab$ by $\{\sqrt{(a)} - \sqrt{(b)}\}^2$.

4. Find the lowest common multiple of the two expressions

$$x^2 + 6x + 5 \text{ and } x^3 - x.$$

5. Reduce to its simplest form the fraction:

$$\frac{m^3 - 6m^2 + 12m - 8}{m^3 - 2m^2 - 4m + 8}.$$

6. Solve the equations:

(1) $\dfrac{5x-7}{3} - \dfrac{x-2}{7} - \dfrac{1-3x}{5} = 2.$

(2) $\left.\begin{aligned} 6x - 4y + z &= 20 \\ 5x - 2y + 2z &= 13 \\ 10x - 5y + 2z &= 30 \end{aligned}\right\}.$

(3) $\dfrac{x-5}{x+5} - \dfrac{x+5}{x-5} = \dfrac{15}{4}.$

7. What is the number which exceeds the sum of its fourth and fifth parts by 33?

8. Divide the number 20 into two such parts that the square of the greater shall exceed the square of the less by 80.

Prove that if any number be divided into two parts, the sum of the squares of these parts is least when the parts are equal.

9. A railway train travels 288 miles at a certain average speed, and if the speed were 4 miles an hour greater, would perform the journey in an hour less. What is the average speed of the train?

10. How is the ratio of two quantities measured algebraically? If a, b, c, d be in proportion, prove that $a^2 + 3b^2$, $a^2 - 2b^2$, $c^2 + 3d^2$, $c^2 - 2d^2$ are also in proportion.

11. Find the sum of 12 terms in an arithmetical progression where the third term is 4 and the seventh term is -4.

MATHEMATICS, SECTION I. 3 hrs.

1. Solve the simultaneous equations:

$$\left.\begin{aligned} 4(x^3 + y^3) &= 7x^2 \\ 2(x^2 + y^2) &= 5x \end{aligned}\right\}.$$

2. Prove that it is impossible to take any number of terms of the progression $1 + \cdot 8 + \cdot 6 + \cdot 4 + \ldots$ starting with the first, so that the sum may be 4.

3. In the expansion of $(a + x)^n$ by the Binomial Theorem, find the greatest coefficient and the greatest term when $a = 8$, $x = \frac{1}{8}$, and $n = 10$.

4. Prove that an angle has but one cosine; but if a decimal between -1 and 1 be presented, it is the cosine of any one of an unlimited number of angles. What are called in the tables "natural cosines"?

Prove that as an angle increases from 0 to 180° its cosine decreases. Is this the case with any other trigonometrical function?

Find to the nearest second the angle greater than two right angles and less than three right angles whose cosine is $-\frac{1}{3}$.

5. Prove that $\sec^2 A = \frac{2 \sec 2A}{1 + \sec 2A}$, and find the angle A, if $\cot A - 3 \cot 3A = 4$.

6. Explain why the value of an angle which is nearly a right angle cannot be safely computed from its given tangent. What method may then be adopted to find the value of the angle?

If $\tan A = 10000$, find the angle A to the nearest second.

7. Find the largest angle of the triangle whose sides are 37, 39 and 24 feet in length, giving the result to the nearest second.

8. A and B are two points 730 yards apart. C is another point, and the angles $CAB = 37° \ 14' \ 26''$ and $CBA = 24° \ 18'$ are measured. Find the perpendicular distance of C from the straight line AB.

9. Given the sides of a triangle, find an expression in terms of these sides for the radius of the circle inscribed in the triangle, also for the distance of the centre of this circle from one of the angular points of the triangle.

10. A regular pyramid of earth is formed on the ground on a square base of which each side is 9 yards long, and the edges of the pyramid are inclined at 21° to the horizon. Find the number of cubic yards of earth in the pyramid.

MATHEMATICS.—SECTION II. 3 hrs.

1. In a right-angled spherical triangle having given the hypotenuse, or side opposite the right angle, 70° 23′ 42″, and another side 48° 24′ 16″, find to the nearest second the angle to which this latter side is opposite.

2. Prove that the three angles of a spherical triangle are never less than two right angles.

If on the surface of a sphere whose radius is 1000 feet the three angles of a triangle exceed two right angles by 9′, compute the area of the triangle.

3. Explain how a line may be represented in position by an algebraic equation.

Find the condition that the straight lines $\frac{x}{a}+\frac{y}{b}=1$, $\frac{x}{c}+\frac{y}{d}=1$, may not intersect one another, and find the perpendicular distance between them.

4. Investigate the equation to the tangent to the parabola $y^2=4a(x+a)$, and find the point of the parabola where the straight line $y=x+2a$ touches it.

5. If perpendiculars are drawn from the foci of an ellipse on the tangent at any point, prove that they meet the tangent in the circumference of the circle whose diameter is the major axis of the ellipse.

6. Differentiate the following expressions with regard to x:

$$(1)\ a(a-x)^{\frac{2}{3}}, \qquad (2)\ \frac{a^3+x^3}{a^2+x^2}, \qquad (3)\ \frac{\tan 2x}{\sin^2 x};$$

and find $\frac{d^2u}{dx^2}$ when $u=a^x \cos x$.

7. Employ Taylor's or Maclaurin's Theorem to expand $\sin 2\theta$ in ascending powers of θ.

8. Find the equation to the tangent to the curve $y^2-3axy+x^3=0$ at any given point, and determine the

point of contact when the tangent is parallel to the axis of x.

9. Integrate the following expressions:

$$(1)\ x\sqrt{(a+x)}\,dx, \quad (2)\ \frac{\sin x\,dx}{\cos 2x}, \quad (3)\ \sqrt{(2x-x^2)}\,dx,$$

and find the value of the last integral between the limits $x=0$ and $x=1$.

10. Prove the formula $\frac{dA}{d\theta}=\frac{1}{2}r^2$, where A is the area between two radii vectores of a curve referred to a pole; and apply this formula to find the area of a parabola cut off by its latus rectum.

11. Apply the Integral Calculus to find the volume of an oblate spheroid.

MECHANICS AND HYDROSTATICS. SECTION III. 3 hrs.

1. Define force. Shew how it is measured in statics. When it is said that a given statical force acts upon a body, what conditions are supposed to be given? How are the terms action and reaction used with reference to statical pressures? When are two bodies said to have the same mass? Is the mass of a body strictly identical with its weight?

2. How can you ascertain the resultant of any number of forces acting at the same point? Prove that if any number of forces acting on a point are in equilibrium, they are parallel in direction and proportional in magnitude to the sides of a polygon.

Four forces acting on a point are represented by four sides of a regular hexagon taken in order, find the magnitude and direction of the resultant.

3. If a substance be weighed on a lever with unequal arms, appearing in one scale to be w_1 lbs., and in the other w_2 lbs., shew that the true weight of the substance is $\sqrt{(w_1 w_2)}$ lbs. If a balance be false, having its arms in the ratio of 15 : 16, at what rate does a customer really pay for a substance which is sold to him from the longer arm at 7*s.* 6*d.* per lb.?

4. Find the centre of gravity of a pyramid with a triangular base.

If a prism and a pyramid have the same base and altitude, find the distance from the base of the centre of gravity of the solid that remains when the pyramid is removed from the prism.

5. On a rough plane inclined at an angle α to the horizon a rough sphere of given weight (w) rests under the action of a given horizontal force (p) applied to it at a given point, find the friction which the plane is exerting on the sphere, and determine whether it acts upwards or downwards.

If the plane be smooth, at what point of the sphere is the force p restricted to act?

6. State the third law of motion, defining the terms used in the statement.

A body weighs 36 lbs., and is moved by a constant pressure which generates in it a velocity of 4 ft. per second, find the weight which the pressure would statically support?

Describe Atwood's machine, and explain its use.

7. A body is projected vertically upwards with a given velocity, find the space it will describe in a given time, neglecting the air's resistance.

A body is projected vertically upwards with a velocity of 192 ft. per sec., and describes 80 ft. in the fourth second, find the measure of the force of gravity.

8. Explain the kind of impulsive action that takes place during the collision of two elastic balls, and shew how it is estimated.

An elastic ball impinges directly on an equal ball at rest, find the velocity of each after impact; and from the expressions obtained, shew that no momentum is lost by the impact, but that vis viva is lost.

9. A closed vessel in the form of a frustum of a cone has horizontal circular ends, whose areas are 7 and 9 sq. inches respectively, and the distance between them is 12 inches. If the vessel is full of water, find the pressure on the lower end in each case as the larger or smaller end is upwards, assuming 1 c. ft. of water to weigh 1000 oz.

10. What distinctive property is there between liquids and gases?

If v be the volume of a gas at a temperature t, v_1 the volume at a temperature t_1, investigate the formula

$$v_1 = \frac{460 + t_1}{460 + t} v.$$

If 200 c. in. of gas, whose temperature is 60° and pressure 30 in., be raised to 280°, and its pressure reduced to 20 in., calculate its volume.

11. Explain the action of the common suction pump, and shew what is the theoretical limit of the height to which water is raised by it.

If the diameter of the piston be 2 inches, and the height of the water in the head of the pump be 24 ft. above the well, what pressure does the piston bear?

INDIAN CIVIL ENGINEERING COLLEGE EXAMINATION.

July, 1873.

ARITHMETIC AND MENSURATION.

1. Add together 5, $3\frac{1}{8}$, $\frac{5}{16}$, $1\frac{5}{24}$, and $\frac{1}{5}$.

2. Subtract $8\frac{7}{17}$ from $10\frac{1}{3}$.

3. Multiply together $5\frac{1}{2}$, $\frac{2}{51}$, $\frac{5}{44}$, $1\frac{1}{16}$, and $\frac{3}{4}$.

4. Divide $7\frac{1}{8}$ by $9\frac{1}{2}$.

5. Add together 19·735, ·000786, 4732·02, and ·375799.

6. Subtract 876·93387 from 974·3216.

7. Multiply 9·238 by 65·4.

8. Divide 2·890721925 by ·03645.

9. Express 2*s*. $7\frac{1}{2}$*d*. as the decimal of £7.

10. Add together $4\frac{1}{5}$, $7\frac{13}{14}$, $9\frac{1}{10}$, $1\frac{1}{35}$.

11. Subtract $34\frac{11}{20}$ from $100\frac{3}{8}$.

12. Multiply together $8\frac{2}{9}$, $45\frac{25}{37}$, $1\frac{1}{13}$, $\frac{81}{130}$, and $2\frac{1}{6}$.

13. Divide $8\frac{4}{21}$ by $5\frac{1}{7}$.

14. Add together 1·45 of a furlong and 7·36 of a yard, and give the answer in feet and the decimal fraction of a foot.

15. Subtract 2·32 of an hour from ·325 of a week.

16. Multiply 380·72 by ·0725.

17. Divide ·7036 by ·73 to 4 places of decimals.

18. Express 1 cwt. 1 qr. 21 lbs. as the decimal of four tons and a half.

N.B.—*The first eighteen questions should be answered before the others are attempted.*

19. Find (by Practice) the dividend on £731. 14*s.* 6*d.* at 14*s.* 2*d.* in the pound.

20. In 5,462,764 square feet how many acres, roods, &c. are there?

21. If 40 men can mow a field of 19 acres in 8½ days of ten hours each, how many acres can 17 men mow in 50 days of 8 hours each?

22. Find the cost of painting the 4 walls of a room at 9*d.* a square yard, the length of the room being 20 ft. 7¾ in., the breadth 15 ft. 4¼ in., and the height 12 ft. 4 in.

23. Multiply by duodecimals 8 ft. 2 in. 4 pts. by 4 ft. 6 in. 9 pts. and the product by 3 ft. 7 in. What does the product become when expressed in cubic feet, cubic inches, and a fraction of a cubic inch?

24. A cubic foot of a certain substance weighs 54 lbs. avoirdupois; find the length of the side of a cube of another substance which weighs 91 lbs. 8 oz. 13½ drs., whose specific gravity is to that of the former as 3 to 2.

25. The area of a rectangular piece of ground is 28 acres and 22 perches. What is its length, its breadth being 625 links?

26. Find the number of gallons of water which pass in 10 minutes under a bridge 17 ft. 8 in. wide, the stream being 10 ft. 11 in. deep, and its velocity 8 miles an hour. [A gallon contains 277·72 cubic inches].

27. The diameters of the top and bottom of a frustum of a cone are 18 inches and 27 inches respectively, and the height is 30 inches; find its volume.

28. If the weight of one cubic foot of water is 62·35 pounds avoirdupois, find the error in calculating the weight of 1,000 cubic feet on the approximate assumption that one cubic foot weighs 1,000 oz.

29. Two opposite angles of a quadrilateral field are together equal to two right angles, and the sides measure respectively $4\frac{1}{2}$ chains, 3 chains 20 links, 2 chains 40 links, and 1 chain 90 links; find its area in acres, roods, perches, &c.

EUCLID, ALGEBRA, AND TRIGONOMETRY (1).

1. Draw from a given point a straight line equal to a given straight line.

2. Prove that if a side of a triangle be produced, the exterior angle is equal to the two interior and opposite angles.

Trisect a given finite straight line.

3. If a straight line be divided into two equal and also into two unequal parts, the squares on the two unequal parts are together double of the square on half the line, and of the square on the line between the points of section.

4. If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the touching line, the centre of the circle shall be in that line.

Describe a circle of given radius which shall touch a given circle and a given straight line.

5. Resolve into factors the expressions:

$$(a+b-c)^2-(a-2b+2c)^2,\quad x^2-19x+88,$$

and $$x^4+x^2y^2+y^4.$$

6. If n be a positive integer, prove that x^n+1 is divisible by $x+1$, and that $x^{2n}+x^{2n-1}+1$ is divisible by x^2+x+1.

7. Determine the highest common factor of

$$3x^3-4x^2-x-6 \text{ and } 2x^4-5x^3+13x^2-13x-18.$$

Find the relation between a, b, c, and d, when the expressions x^2+ax+b and x^2+cx+d have a common factor.

8. Solve the equations:

(1) $(x-a)^3+(x-b)^3+(x-c)^3=3(x-a)(x-b)(x-c)$.

(2) $16x^2-192x+551=0$.

(3) $\dfrac{y+z-x}{b+c}=\dfrac{z+x-y}{c+a}=\dfrac{x+y-z}{a+b}=a+b+c$.

9. Find an expression for the sum of n terms of an arithmetic series, having given the first two terms.

Sum the series:

$$1+4+7+10+\ldots \text{ to } n \text{ terms},$$

$$1^3+2^3+3^3+\ldots+n^3,$$

and $$2^2\times1+3^2\times2+4^2\times3+\ldots+n^2\times(n-1).$$

10. Define the tangent of an angle, and trace the variations in its sign and magnitude as the angle increases from zero to four right angles.

Trace the changes in sign of the expression $+\tan 3\theta \tan 4\theta$, as θ changes from 0 to π.

11. Prove by means of geometrical figures the formulæ:

$$\cos(A-B)=\cos A\cos B+\sin A\sin B,$$

and $$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}.$$

12. Find an expression for all the angles which have a given tangent.

Having given $\tan A$, find $\tan \frac{A}{2}$, and explain, by means of a figure, why two values are obtained.

Prove that $\tan(37^\circ\ 30') = \sqrt{(6)} + \sqrt{(3)} - \sqrt{(2)} - 2$.

13. Find expressions, in terms of the sides of a triangle for the radii r and R, of the inscribed and circumscribed circles.

Prove that the area of the triangle is equal to

$$Rr(\sin A + \sin B + \sin C).$$

Prove also that the distance between the centres of the inscribed and circumscribed circles is equal to

$$\sqrt{(R^2 - 2Rr)}.$$

EUCLID, ALGEBRA, AND TRIGONOMETRY (2).

1. Inscribe a circle in a given triangle.

Also describe a circle touching one side and the other two sides produced.

Two sides of a triangle of given perimeter are given in position, prove that the third side always touches a certain circle.

2. Inscribe a circle in a given equilateral and equiangular pentagon.

3. If the exterior angle of a triangle, made by producing one of its sides, be bisected by a straight line which also cuts the base produced, the segments between the dividing straight line and the extremities of the base shall have the same ratio which the other sides of the triangle have to one another.

Find the locus of a point at which two given circles subtend equal angles.

4. Equal parallelograms which have one angle of the one equal to one angle of the other have their sides about the equal angles reciprocally proportional.

5. Explain what are meant by scales of notation, and shew how to express any whole number, or any fraction, in a proposed scale.

Express 33·49 in the scale of which 7 is the base to 5 radix-places, and multiply the result by 2·5 in the same scale.

6. Prove that the number of combinations of n things taken r together is equal to the number of combinations of n things taken $n-r$ together.

A bag contains n different coins; how many different combinations of heads and tails can be obtained by placing them on a table, 1st, 2 at a time, 2nd, r at a time?

7. Write down the coefficients of x^r in the expansions of $(1+x)^n$ and $(1-x)^{-n}$.

Find the coefficients of x^r in the expansions of

$$\frac{4x-1}{(x-1)^2(x+2)} \text{ and } (x+x^2+x^3+\dots ad\ inf.)^2.$$

8. Find the chance of throwing 6 with two dice.

What are the odds against throwing 6 in three throws with two dice?

9. Shew that one solution, in positive integers, of the equation

$$ax - by = c,$$

can always be found; and hence find the general solution in positive integers.

Investigate expressions for the positive integral values of x and y in the equation

$$5x + 11y = 1,$$

and find all the positive integral solutions of the equation

$$xy + 2y + x = 18.$$

10. Having given two sides and the included angle of a triangle, shew how to find the remaining parts by help of a table of logarithms.

At the top C of a tower, of height h, the angles of depression of two objects A, B, on the horizontal plane on which the tower stands are $\frac{1}{4}\pi - \alpha$ and $\frac{1}{4}\pi + \alpha$ respectively, and the angle subtended by the objects is 2α; prove that $AB = 2h \tan 2\alpha$, and that the angles CAB and CBA are $\frac{1}{4}\pi - \alpha$ and $\frac{1}{4}(3\pi) - \alpha$.

11. Sum the series, each to n terms:

(1) $\cos\alpha + \cos 2\alpha + \cos 3\alpha + \ldots.$

(2) $\tan\alpha + 2 \tan 2\alpha + 2^2 \tan 2^2\alpha + \ldots.$

12. Enunciate and prove Demoivre's Theorem.

Find all the roots of the equation

$$x^{2n} - 2x^n \cos n\theta + 1 = 0.$$

13. If n be an even integer, investigate an expression for $(\sin\theta)^n$ in a series of cosines of multiples of θ.

STATICS.

1. How is a statical force measured numerically? How is it represented geometrically? Give examples of the composition and resolution of force. If a force act upon a point, illustrate, by a figure, how the single force may be replaced (1) by two equal forces acting on the point, (2) by three equal forces acting on the point. In the latter case, if the single force be 20 lbs., assign a numerical value to each of the three equal forces as represented.

2. State and prove the condition of equilibrium on a straight lever acted on by any two forces, the fulcrum being placed between the points of application of the forces, and find the pressure on the fulcrum in a direction perpendicular to the lever.

A straight horizontal lever without weight is 18 inches long, a force of 36 ounces, which makes an angle of 150° with the lever at one extremity, balances a weight of 9 ounces at the other extremity; find the place of the fulcrum and the pressure upon it perpendicular to the lever.

3. Define the centre of gravity of a body or system.

Shew, in the case of three heavy particles, that they will have a centre of gravity, and only one centre of gravity. The three particles being equal and in a straight line, find the distance of their common centre of gravity from any point on the line; and if A, B, C be the positions of the particles and (G) their centre of gravity, prove

$$AB^2 + AC^2 + BC^2 = 3\,(AG^2 + BG^2 + CG^2).$$

4. Describe the common steelyard, and shew how to mark the graduations on the steelyard to which the following data belong: the moveable weight is 1 lb., the weight of the steelyard 1 lb., the distance of the fulcrum from the fixed point where the substance to be weighed is suspended is 3 in., and the distance of the centre of gravity of the beam from the same point is 4 in.

5. When does a machine act at a mechanical advantage? Take any of the simple mechanical powers and explain why the practical advantage differs from the theoretical advantage. In the common wheel and axle, find the equilibrium relation of the power and the weight.

The radius of the wheel is four times that of the axle, and the string on the wheel can just sustain a tension of 48 lbs.; find the greatest weight that can be raised by the machine.

6. When a weight is placed on an inclined plane what is the direction of the resistance of the plane

(1) when it is smooth, (2) when it is rough and the weight is in a state bordering on motion down the plane?

The height of an inclined plane is to its length as 3 to 5, what is the coefficient of friction or the limiting angle of resistance when any weight of a given substance can be just supported on such a plane? If the weight resting on this plane be 30 lbs., what is the least force which, acting along the plane, can just draw the weight up the plane?

7. State the principle of "virtual velocities" as applicable to the simple machines, and prove it in the case of a horizontal straight lever when weights are suspended at its extremities.

If P and W be the weights, h and h_1 the vertical heights through which P and W are moved, is it true, when the angle is not small through which the lever is turned, that in this case $Wh_1 = Ph$?

The following questions do not form part of the obligatory *examination. The answers to them should be sent up in a separate book.*

DYNAMICS.

1. How is the force of gravity at the earth's surface estimated? Is it the same for all positions on the earth? If not, assign any causes for its variation. If 32 be taken as the measure of gravity, what units are referred to, and how is this measure related to the space fallen through by a heavy particle in one second? In the case of a body projected in the direction of gravity, assuming the relations between the velocity and the time, and the space and the time, find the expression for the velocity in terms of the space.

Two bodies are projected at the same instant from opposite ends of the same vertical line, each with the velocity that would be acquired in falling down the line; find where the bodies will meet.

2. Shew how to find the range and greatest height of a projectile in vacuo.

Find the velocity and angle of projection of a projectile that will strike a balloon horizontally, the balloon being at the vertical height of 400 ft., and its horizontal distance from the mouth of the gun being $800\sqrt{(3)}$ ft.

3. State the law of motion which connects the statical and dynamical measures of force, and shew how weight (W) is estimated when expressed by the relation $W = Mg$, (M) being the mass of the body. If one heavy body descend drawing up another heavy body by means of a perfectly flexible string over a fixed pulley, shew how to find the accelerating force and the tension of the string. Find the numerical value of the tension when the weights are 20 and 10 ounces respectively.

4. How is impulsive force estimated? When one elastic ball strikes another in direct impact, how is the whole motion before impact invariably related to the whole motion after impact?

A ball ($3m$) impinges directly with a velocity of 30 feet per second on a ball ($5m$) at rest. The modulus of elasticity being $\frac{3}{5}$, determine how the motion of each ball is affected by the impact.

5. If (t) the time of a small oscillation of a simple pendulum be expressed by the relation $t = \pi\sqrt{\left(\frac{l}{g}\right)}$, what do the symbols express, and to what units are they referred?

A pendulum which oscillates seconds at the equator would, if carried to the pole, gain 5 minutes a day; compare the force of gravity at the equator with the force of gravity at the pole.

6. If a heavy body revolve uniformly with a velocity (v) in a circle, radius (r), explain what is meant by

the centrifugal force, and shew how $\frac{v^2}{r}$ expresses the measure of this force.

A stone of 8 pounds in weight is whirled round uniformly in a horizontal plane by a string of a yard and a half long, one end of the string being fixed; find the number of revolutions made in one minute when the tension of the string is 12 lbs.

PURE MATHEMATICS. I.

1. Shew how the position of a point in a plane is determined by its coordinates referred to rectangular axes.

If the coordinates of P be -1 and 2, and those of $Q + 3$ and -1, find the length of PQ and its inclination to the axis of x.

2. Prove that $y = mx + c$ is the equation of a straight line, giving the geometrical meaning of m and c.

Write down any other forms of the equation of the straight line, stating the geometrical meaning of the constants involved in each case.

3. Find the perpendicular distance of the point (h, k) from the straight line

$$Ax + By + C = 0.$$

4. Find the equations to the two straight lines which bisect the angles between the straight lines

$$x + y - a = 0 \text{ and } x - y + a = 0.$$

5. Find the equation of the circle referred to any pair of rectangular axes.

Apply this equation to prove that the angles in the same segment are all equal to one another.

6. Find the equation of the tangent at any point P of the parabola whose equation is $y^2 = 4ax$, and deduce that of the normal.

If the normal at P meet the parabola again in Q, and QR be drawn parallel to the axis to meet the double ordinate PP' produced in R, prove the rectangle $PP' \times P'R$ is constant.

7. Prove that the locus of the middle points of a set of parallel chords to an ellipse is a diameter of the ellipse, and that the tangents at the extremities of this diameter are parallel to the bisected chords.

8. Prove the existence and find the equations of a pair of asymptotes to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

9. Prove the equation

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1,$$

where α, β, and γ are the inclinations of any straight line to three rectangular axes.

10. Define the locus of an equation between three rectangular coordinates x, y, and z. Interpret the following equations:

(1) $x^2 + y^2 + z^2 = 0$. (2) $x^2 + y^2 + z^2 + a^2 = 0$.

(3) $x^2 + (y-a)^2 = 0$. (4) $x^2 - a^2 = 0$.

11. Find any form of the equations of a straight line in space referred to three rectangular axes, giving the geometrical meaning of the involved constants.

12. Find the equation of a plane referred to three rectangular axes.

The planes

$$2x + 3y + 4z = a,$$
$$5x + 4y + 3z = b,$$

intersect the plane $x = c$; find the equations to the two lines of intersection and the cosine of the angle between them.

13. Interpret the equations:

(1) $\begin{cases} y = mx \\ z = c. \end{cases}$

(2) $(x-a)^2+(y-b)^2+(z-c)^2=c^2.$

(3) $$(x-a)^2+(y-b)^2+(z-c)^2 = k^2\{l(x-a)+m(y-b)+n(z-c)\}^2.$$

(4) $(x-a)^2+(y-b)^2+(z-c)^2=\{lx+my+nz\}^2,$

where $l^2+m^2+n^2=1$ in 3 and 4.

Prove that the surfaces in 3 and 4 intersect in plane curves, and find the equations of the planes of intersection.

14. Prove that every equation of an odd degree has at least one real root.

15. *State* the rule for expressing the coefficients of an equation in terms of the roots, and deduce the value of the sum of the reciprocal of the roots.

Candidates are at liberty to substitute any of the following questions for an equal number of the foregoing.

A. From a point P tangents are drawn to a parabola. If the chord of contact meet the axis in Q, and PQ be always a mean proportional between AQ and the distance of the focus from the directrix, prove that the locus of P is an ellipse.

B. Find the locus of a point such that the difference of the squares of its shortest distances from two given non-intersecting straight lines is constant.

C. If any group consisting of an even number of terms is wanting in an equation, there are at least as many imaginary roots as there are missing terms.

PURE MATHEMATICS. II.

Logarithm Books will be supplied.

1. Prove that any two sides of a spherical triangle are together greater than the third side, and the three sides of a spherical triangle are together less than the circumference of a great circle.

2. Given the three angles of a spherical triangle, find one of the sides of the triangle in a form adapted to logarithmic computation.

Find a side of the equilateral spherical triangle of which each angle is 120°.

3. In reference to a spherical triangle, what is the "spherical excess?"

The area of an equilateral spherical triangle is equal to the area of a great circle of the sphere, find an angle of the spherical triangle.

4. In a spherical triangle, A, B, C being the angles, and a, b, c the sides subtending them respectively, prove

$$\tan\frac{A+B}{2} = \frac{\cos\frac{a-b}{2}}{\cos\frac{a+b}{2}}\cot\frac{C}{2}.$$

Write down the corresponding formula for $\tan\frac{A-B}{2}$.

Given $a = 81° \, 17'$, $b = 59° \, 13'$, $C = 125° \, 36'$, find A and B.

5. Define a differential coefficient, and find from definition the differential coefficient of a^x.

Differentiate the following functions, and in each case express the result in its simplest form.

(1) $y = \log_e\{x + \sqrt{(1 + x^2)}\}$.

(2) $y = \frac{(\tan x)^3}{3} - \tan x + x$.

(3) $y = \tan^{-1}\frac{2x}{1 - x^2}$.

If $y = \varepsilon^{-x} \cos x$, prove $\frac{d^4y}{dx^4} + 4y = 0$.

6. Apply the theorem, known as Maclaurin's, to expand $\cos x$ in terms of (x) as far as the term involving x^4; and given $y^2 - yx - 1 = 0$, shew by the same theorem that $y = \pm 1 + \frac{1}{2}x \pm \frac{1}{2^2} \cdot \frac{x^2}{1.2}$ expresses the three first terms of the development of y in terms of x.

7. If $y = f(x)$ and for a particular value of x, $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} = 0$, investigate the conditions that for that value of x, y may have a maximum or minimum value.

Inscribe in a given circle the greatest isosceles triangle, and shew by the differential test that the triangle is a maximum.

8. Given $y = f(x)$, the equation to a plane curve, and $y - y' = a(x - x')$, the equation to the normal at a point $(x'y')$ of the curve, obtain the value of (a).

In the parabola prove that the normals to the curve drawn at opposite extremities of a focal chord are at right angles to each other.

9. What is a point of inflexion of a plane curve? Shew generally how to determine from its equation the points of inflexion of a curve?

Ex. $$y = \frac{2a}{x}\sqrt{(2ax - x^2)}.$$

10. If $r = f(\theta)$ be the polar equation to a curve, shew how to obtain from this the equation between (p) and (r) where (p) is the perpendicular from the pole upon the tangent.

Obtain this equation when $r = ae^{m\theta}$, and shew that in this curve the radius vector makes a constant angle with the tangent at its extremity.

11. Assuming the expression for the radius of curvature at any point of a polar curve, shew how the evolute of the curve may be found, and prove that the evolute of the curve $r = ae^{m\theta}$ is similar to the curve itself.

12. Integrate the following functions.

$$(1)\ \int\frac{2adx}{a^2-x^2}.\quad (2)\ \int\frac{x^2dx}{\sqrt{(a^2-x^2)}}.\quad (3)\ \int dx\,(\tan x)^2.$$

Any of the following questions may be substituted for an equal number of the above, it being stated in each case for which question the substitution is made.

(A) Shew that when the radius of curvature of a plane curve has a maximum or minimum value the circle of curvature has a contact of the third order with the curve.

Determine the radii of maximum and minimum curvature in an ellipse.

(B) How are the epicycloid and hypocycloid curves described? Obtain equations to *either* of these curves needed to find the length of the curve, and determine the length corresponding to one revolution of the generating circle.

(C) Shew that the whole surface (S) of a prolate spheroid, generated by the revolution of an ellipse about its major axis, is expressed by the equation

$$S = \frac{2\pi ab}{e}\{\sin^{-1}e + e\sqrt{(1-e^2)}\},$$

where (a) and (b) are the semi-axes, and $(a^2 - b^2) = a^2e^2$.

MIXED MATHEMATICS.

1. Define a couple, in statics, and find the resultant of two couples acting in different planes.

Find the direction and magnitude of the axis of the couple which is the resultant of three equal couples acting in three planes at right angles to each other.

2. Investigate the conditions of equilibrium of a system of forces acting on a rigid body in one plane.

A heavy rod *AB*, which is moveable about the end *A*, rests with the end *B* on a smooth inclined plâne; find the direction and magnitude of the action at the end *A*.

3. Obtain formulæ for determining the position of the centre of gravity of an area defined by polar coordinates.

Find the centre of gravity of a semicircular lamina; first, when it is homogeneous; second, when its density at any point varies as the distance of that point from the centre of the circle.

4. Define a catenary, and prove that, if c be the length of chain, the weight of which is equal to the tension at the lowest point (A), and if s be the length of any arc AP, and ϕ the inclination to a horizontal plane of the tangent at P,

$$s = c \tan\phi.$$

Shew hence, or otherwise, that the equation to the catenary in rectangular coordinates is

$$y = \frac{c}{2}\left(\varepsilon^{\frac{x}{c}} + \varepsilon^{-\frac{x}{c}}\right).$$

A heavy chain is suspended from two points in the same horizontal plane; prove that there is a certain length of chain for which the tension at either point of support will be a minimum.

5. A point moves in a straight line, and its distance, at a time t, from a fixed point in the line is x; prove that its acceleration is $\frac{d^2x}{dt^2}$.

Determine the motion of a particle, initially at rest, which is acted upon by a force to a fixed point, varying directly as the distance from that point.

6. Investigate expressions for the tangential and normal components of the acceleration of a point moving in a plane curve.

A heavy particle moves on the arc of a smooth cycloid having its axis vertical and vertex downwards; determine its motion, and prove that, if the particle start from a cusp, the pressure on the curve at the vertex is twice the weight of the particle.

7. Define the term "vis viva," and prove that, if two imperfectly elastic balls impinge directly on each other, vis viva is lost by the impact.

State what becomes of the lost vis viva.

8. A perfectly elastic particle is let fall, in a resisting medium, from a given height above a horizontal plane; determine how far it will rise after striking the plane, the force of resistance being proportional to the square of the velocity.

9. Define a fluid, and explain how fluid pressure is measured.

Distinguish between gases and liquids, and prove that, in a heavy liquid at rest, the pressure is the same at all points in the same horizontal plane.

10. Shew how to find the resultant vertical pressure of a heavy liquid on any surface with which the liquid is in contact. A hollow cone, having its axis vertical and vertex upwards, is just filled with water; compare the resultant vertical pressures on its base and curved surface.

11. Find the conditions of equilibrium of a heavy body floating partially immersed in a liquid.

Prove that an isosceles triangular lamina can float with its base vertical in a liquid of double its own density.

12. Describe the construction and use of a Diving Bell.

Does the tension of the supporting chain increase or decrease as the Diving Bell descends?

13. State the experimental law connecting the pressure (p) and density (ρ) of a gas when the temperature is unchanged, and also the experimental law connecting the density and temperature (t) when the pressure is unchanged; and deduce from these laws the relation

$$p = k\rho\,(1 + \alpha t).$$

A spherical envelope, filled with gas at a temperature zero, is compressed into a sphere of half its original radius, and the temperature is raised to 100° C; compare the new pressure of the gas with its original pressure.

Candidates are at liberty to substitute any of the following for any of the preceding questions.

A. Prove that any system of forces in space can be reduced to a single force and a couple in a plane perpendicular to the direction of the force.

B. A fine string just surrounds a circle, and every element of the string is acted upon by a force to the middle point of a given radius, proportional to the distance from that point; find the tension, and the pressure on the circle, at any point.

C. Investigate the equation for the motion of a particle under the action of a central force,

$$\frac{d^2u}{d\theta^2} + u = \frac{P}{h^2u^2}.$$

Having given that $P=\mu u^2$, and that the particle is projected, at the initial distance a, with the initial velocity $\sqrt{\frac{\mu}{a}}$, determine the orbit.

D. A square lamina floats in a liquid with its plane vertical, and with one angular point beneath the surface; determine its position of equilibrium.

E. Define the metacentre, and investigate a formula for determining its position in the case of a solid of revolution floating with its axis vertical.

A solid cone floats with its axis vertical and vertex downwards; find the condition of stability.

EXAMINATION FOR FIRST APPOINTMENTS TO THE CAVALRY AND INFANTRY.

January, 1874.

EUCLID (BOOK I.)

1. If two angles of a triangle be equal to one another, the sides also which subtend, or are opposite to, the equal angles shall be equal to one another.

2. If from the ends of the side of a triangle there be drawn two straight lines to a point within the triangle, these shall be less than the other two sides of the triangle, but shall contain a greater angle.

3. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are together equal to two right angles.

4. Prove that the angle between the bisectors of two consecutive angles of a quadrilateral is half the sum of the two remaining angles.

5. If the square described on one of the sides of a triangle be equal to the squares described on the other two sides of it, the angle contained by these two sides is a right angle.

6. If any point P be joined to A, B, C, D, the four angular points of a rectangle, prove that the squares on PA and PC are together equal to the squares on PB and PD.

ARITHMETIC.

1. Add together $\frac{4}{7}$, $1\frac{5}{6}$, $\frac{8}{21}$, and $\frac{1}{42}$.

2. Subtract $6\frac{4}{5}$ from $8\frac{3}{7}$.

3. Multiply $4\frac{1}{5}$ by $\frac{10}{63}$.

4. Divide $1\frac{7}{8}$ by $3\frac{5}{12}$.

5. Add together 70·152, ·00628, 1·905, and 3281·6013.

6. Subtract 47·6125 from 350·013.

7. Multiply 501·32 by ·0063.

8. Divide 17·465 by 98·6 to three places of decimals.

9. Find the value of 6·125 of £1. 13*s.* 4*d.*

10. In 3706529 inches how many miles, furlongs, poles, &c. are there?

11. If 69 lbs. of salt cost 7*s.* $2\frac{1}{4}$*d.*, what will be the value of 4 cwt. 1 qr. 7 lbs.?

12. Find the simple interest on £5,781 for 10 years at $4\frac{1}{2}$ per cent. per annum.

13. Add together $5\frac{2}{9}$, $3\frac{1}{12}$, and $1\frac{4}{27}$.

14. Subtract $7\frac{8}{11}$ from $10\frac{12}{143}$.

15. Multiply together $10\frac{1}{8}$, $\frac{6}{13}$, $\frac{26}{27}$, and $1\frac{1}{3}$.

16. Divide $6\frac{4}{9}$ by $2\frac{4}{63}$.

17. Add together 1·375 gallons and 3·5 quarts.

18. Subtract 33·75 of a minute from 1·95 of an hour.

19. Multiply ·401296 by 8·05.

20. Divide 2054·95 by ·0563.

21. Express 7*s.* $9\frac{3}{4}$*d.* as the decimal of £8.

22. In 57625318 square inches how many acres, roods, &c., are there?

23. A person after paying an income-tax on £515. has £484. 19*s.* 7*d.* left, how much did he pay in the £?

24. A bankrupt's estate amounts to £910. 3*s.* $1\frac{1}{2}$*d.*, and his debts to £1,875., what can he pay in the £?

25. At what rate per cent. will £425. amount to £573. 15*s.* in 5 years at simple interest?

26. Find by practice the value of 5cwt. 3qrs. 21lbs. at £1. 5*s.* 8*d.* per cwt.

27. A person completes a journey of 1,056 miles in 12 days, travelling 11 hours a day; in how many days would he complete 480 miles, going 6 hours a day at the same rate?

MATHEMATICS.

(ALGEBRA, MENSURATION, LOGARITHMS).

Logarithm books will be supplied.

1. Multiply the factors $(x-3)$, $(x+3)$, $(x-5)$, and find the values of the product when $x=5$, and when $x=9$.

Find the numerical value of the expression

$$\{2^5+(49)^{\frac{1}{2}}-(64)^{\frac{2}{3}}\}.$$

2. Perform the operations indicated in the following examples, and in each case express the result in its simplest form:

(α) $(x^3-2x^2-23x+60)\times(x+1)^2$.

(β) $\{x-2+\sqrt{(3)}\}\times\{x-2-\sqrt{(3)}\}$.

(γ) $(8x^3-36x^2+54x-27)\div(2x-3)$.

(δ) $\dfrac{3a+4b}{3a-4b}-\dfrac{3a-4b}{3a+4b}$.

3. Reduce to their lowest terms:

(α) $\dfrac{6x^2-13xy+6y^2}{6x^2+5xy-6y^2}$ (β) $\dfrac{(2a-3b)^2-(a+2b)^2}{(3a-b)^2}$.

4. Find the algebraical expression which is called the least common multiple of

$$(1-x^2),\ (1-x^3),\ \text{and}\ (1+x)^2.$$

5. If $\frac{a}{b}=\frac{b}{c}$, express the relation between a, b, c in the form of a proportion.

Given $\frac{a+b}{a-b}=\frac{c+d}{c-d}$, prove $a:b::c:d$.

Given $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, prove

$$ac:bd::a^2+c^2+e^2:b^2+d^2+f^2.$$

6. Find the value of $\frac{\sqrt{(3)}+\sqrt{(2)}}{\sqrt{(3)}-\sqrt{(2)}}$ to three decimal places.

Prove $\frac{1+2^{\frac{2}{3}}+2^{\frac{4}{3}}}{3+2^{\frac{1}{3}}+2^{\frac{2}{3}}}=\frac{1+2^{\frac{1}{3}}}{1+2^{\frac{2}{3}}}$.

7. Solve the following equations:

$$(\alpha)\quad \frac{x+1}{7}-\frac{2x-6}{9}+4=\frac{x+15}{63}.$$

$$(\beta)\quad \frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{x}{6}.$$

$$(\gamma)\quad \left.\begin{aligned} 1+x+y &= \frac{4x+3y}{3}-(4x-3y) \\ \frac{x+y}{9} &= y-x \end{aligned}\right\}.$$

$$(\delta)\quad 6x^2-x=1.$$

$$(\varepsilon)\quad \frac{x^2-x+1}{x^2+x-2}=\frac{3}{4}.$$

$$(\zeta)\quad \left.\begin{aligned} \frac{x^2}{y^2}+\frac{y^2}{x^2} &= \frac{17}{4} \\ x^2-y^2 &= 48 \end{aligned}\right\}.$$

Form a quadratic equation whose two roots shall be

$$-3+2\sqrt{(-1)},\ -3-2\sqrt{(-1)}.$$

8. A grocer finds that by mixing black and green tea together in the ratio of two pounds of black to one of green he can sell the mixture without loss at 3*s.* 3*d.* a pound, but if the mixture is made in the ratio of seven pounds of black to two of green he can sell at 3*s.* 2*d.* a pound; what was the cost per pound of the black and green tea respectively?

9. A grazier offered to sell a flock of sheep for £262. 10*s.*, but before a purchase was completed 10 of the sheep died, and it was found that the remainder of the sheep must be each sold at an advance of 12*s.* 6*d.* in order that the grazier may receive the same sum by the sale; how many sheep were there in the flock?

10. *A* and *B* have each the same sum of money: *A* buys equal amounts of 3 per cent. stock at 91 and of $3\frac{1}{4}$ per cent. stock at $97\frac{1}{2}$; *B* invests his money equally in the purchase of the same stocks; *A*'s income being one shilling more than *B*'s; how much money had each?

11. In any system of logarithms shew that $\log 1 = 0$, and that the logarithms of all fractions less than unity are negative.

Explain why to a base 10 the decimal parts of the logarithms of the numbers 500, 5, ·005 may be expressed by the same digits; write down these logarithms, it being given that $\log_{10} 8 = {\cdot}9030900$.

Find by means of the tables $({\cdot}005718)^{\frac{2}{5}}$.

12. The greatest angle of a parallelogram is 125° 30′, the sides containing the angle are 95·5 and 46·5 feet respectively; find the area of the parallelogram in square feet.

13. Given one side of an equilateral triangle, find its area. Hence find the area of an equilateral and equiangular hexagon inscribed in a circle whose radius is 10 feet.

14. A uniform ditch is 100 yards long and 4 feet in perpendicular depth; a vertical section of the ditch is a trapezoid, whose opposite parallel sides are 3 feet and 1 foot 8 inches respectively; find the number of tons of water the ditch would contain, it being given that a cubic foot of water weighs 62·5 pounds.

MATHEMATICS.

(GEOMETRY AND TRIGONOMETRY).

Logarithm books will not be required.

1. If a straight line be divided into any two parts, the squares on the whole line, and on one of the parts, are equal to twice the rectangle contained by the whole and that part, together with the square on the other part.

2. Describe a square that shall be equal to a given rectilineal figure.

If a square and a rectangle have equal areas, prove that the perimeter of the square is less than that of the rectangle.

3. If two circles touch one another internally, the straight line which joins their centres, being produced, shall pass through the point of contact.

4. The angle at the centre of a circle is double of the angle at the circumference on the same base, that is, on the same arc.

AB and *CD* are two chords in a circle which intersect when produced in the point *E* without the circle, prove that the difference of the angles subtended at the centre by the arcs *AC* and *BD* is double of the angle *AEC*.

5. In a given circle, inscribe a triangle equiangular to a given triangle.

6. Describe a circle about a given equilateral and equiangular pentagon.

7. The sides about the equal angles of triangles which are equiangular to one another are proportionals; and those which are opposite to the equal angles are homologous sides, that is, are the antecedents or the consequents of the ratios.

The diameter AB of a circle is produced to C, and through C the line CD is drawn at right angles to the line ABC; prove that if the straight line AD cuts the circle in E, the rectangle AE, AD will be equal to the rectangle AB, AC.

8. Parallelograms which are equiangular to one another have to one another the ratio which is compounded of the ratios of their sides.

9. $ABCD$ is a rectangle, and E, F, G, and H are the feet of the perpendiculars let fall upon the diagonals from the angular points; prove that $EFGH$ is also a rectangle.

10. Define the circular measure of an angle.

If the number of degrees in an angle be equal to the number of grades in the complement of the same angle, prove that the circular measure of the angle is $\frac{5\pi}{19}$.

11. Define the principal trigonometrical ratios, and express each of them in terms of the tangent.

Find $\sin 60°$ and $\cot 30°$.

12. Find $\sin(A+B)$ in terms of the sines and cosines of A and B, stating the limitations in the values of A and B assumed in the proof. Apply to find $\sin 75°$.

13. Find $\tan 2A$ and $\sin 3A$.

14. Given $\tan\frac{A}{2} = \frac{a}{b}$, find $\sin A$ and $\sin 2A$.

15. In any triangle, of which the semi-perimeter is s, prove the formula $\sin\frac{A}{2} = \sqrt{\left\{\frac{(s-b)(s-c)}{bc}\right\}}$.

16. From B and A the acute angles of the right-angled triangle ABC the lines BD and AE are drawn to the opposite sides of the triangle such that the angles DBC and EAC are each equal to α.

Prove that $DE = AB \tan\alpha$.

17. Given $A = 40°$, $a = 140{\cdot}5$, $b = 170{\cdot}6$. Find B and C.

Given that $\log\sin 40° = 9{\cdot}8080675$.
$\log 1405 = 3{\cdot}1476763$.
$\log 1706 = 3{\cdot}2319790$.
$\log\sin 51° \, 18' = 9{\cdot}8923342$.
$\log\sin 51° \, 19' = 9{\cdot}8924354$.

18. Two sides of a triangle are 85 and 75 yards respectively, and the included angle is 70°; find the remaining angles.

Given that $\log 160 = 2{\cdot}20412$.
$\log\cot 35° = 10{\cdot}1547732$.
$\log\tan 5° \, 6' = 8{\cdot}9505967$.
$\log\tan 5° \, 7' = 8{\cdot}9520211$.

EXAMINATION FOR ADMISSION TO THE ROYAL MILITARY ACADEMY, WOOLWICH.

January, 1874.

ARITHMETIC AND LOGARITHMS.

1. Add together $2\frac{7}{9}$, $1\frac{3}{13}$, and $\frac{19}{52}$.

2. Subtract $10\frac{5}{39}$ from $12\frac{2}{9}$.

3. Multiply together $1\frac{94}{109}$, $12\frac{1}{9}$, $5\frac{13}{19}$, $\frac{57}{58}$, and $\frac{1}{2}$.

4. Divide $11\frac{12}{13}$ by $2\frac{51}{52}$.

5. Add together 51·81035, 3·78109, ·0013, 5·1005763, and ·0000037.

6. Subtract 43·86165 from 554·6103.

7. Multiply 6·0035 by ·016.

8. Divide 60·105311 by ·0035.

9. Reduce £2. 9*s*. 6*d*. to the decimal of £4. 2*s*. 6*d*.

10. Add together $\frac{4}{13}$ of $\frac{52}{57}$, $\frac{19}{23}$ of $7\frac{2}{3}$, and $2\frac{15}{19}$ of $\frac{17}{106}$.

11. Subtract $199\frac{5}{76}$ from $201\frac{1}{19}$.

12. Multiply together $8\frac{13}{15}$ of $1\frac{6}{19}$, $\frac{3}{35}$ of $1\frac{1}{11}$, and $\frac{22}{27}$ of 9.

13. Divide $6\frac{1}{3}$ by $1\frac{8}{11}$.

14. Add together 15·21435 of a day and 3·6105 of an hour, and give the answer in minutes and the decimal fraction of a minute.

15. Subtract 50·14 of a pole from 1·6105 of a mile, and give the answer in furlongs and the decimal fraction of a furlong.

16. Multiply 69·302 by 1·0093.

17. Divide 419·013 by 6·013 to three places of decimals.

18. Reduce 9·72 grains to the decimal of a pound troy.

19. Reduce 6 miles 7 furlongs 4 yards 2 feet to inches.

20. If 17 cwt. 1 qr. of sugar cost £24. 3*s.*, what will be the price of 15 lbs.?

21. Find (by Practice) the dividend on £3,105. 10*s.* at 14*s.* 7½*d.* in the pound.

22. Find the simple interest on £1,576. 10*s.* for 8 years at 6¼ per cent. per annum.

23. In 3724816 ounces how many tons, cwts., &c.?

24. If 36 men can mow a field of 15 acres in 1 day, how long will it take 6 men to mow one of 7 acres 5 square poles?

25. Find (by Practice) the value of 42 oz. 8 dwts. 12 grs. of silver at 5*s.* 10*d.* per oz.

26. Find the compound interest on £6,530 in 3 years at 6 per cent. (neglecting fractions of a penny).

27. Compute by means of the tables the value of $89125 \times \sqrt[5]{\left\{\frac{(7\cdot018)^8}{(4\cdot01567)^{16}}\right\}}$ to three places of decimals.

28. Given

$$\log 3\cdot1 = \cdot4913617 \text{ and } \log 6\cdot122774 = \cdot7869483,$$

find the value of $31^{-\frac{1}{7}}$.

29. Extract the square root of $14\frac{350}{2023}$; also the square root of 19 accurately to six places of decimals.

30. How much money must be invested in the 3 per cent. consols at 93⅞ so as to yield an income of £576 per annum?

EUCLID.

1. If one side of a triangle be produced the exterior angle shall be greater than either of the interior opposite angles.

2. In any right-angled triangle, the square which is described on the side subtending the right angle is equal to the squares described on the sides which contain the right angle.

3. If ACB be a triangle right-angled at C, and if O be the point of intersection of the diagonals of the square on AB, prove that CO bisects the angle ACB.

4. Describe a square that shall be equal to a given rectilineal figure.

Of all rectangles with the same perimeter, prove that the square has the greatest area.

5. If a straight line drawn through the centre of a circle bisect a straight line in it which does not pass through the centre, it shall cut it at right angles; and if it cut it at right angles it shall bisect it.

6. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

From the point A in the circumference of the circle $APQB$ three chords AP, AQ, and AB are drawn so that AB bisects the angle between PA and QA produced.

Prove that the chords BP and BQ are equal.

7. If from any point without a circle two straight lines be drawn, one of which cuts the circle, and the other touches it; the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, shall be equal to the square on the line which touches it.

8. Inscribe a circle in a given triangle.

If P and Q be the centres of the circle inscribed in ABC, and of the circle touching the side BC and the other two sides produced, prove that the points B and C lie upon the circle of which PQ is the diameter.

9. If two triangles have one angle of the one equal to one angle of the other, and the sides about the equal angles proportionals, the triangles shall be equiangular to one another, and shall have those angles equal which are opposite to the homologous sides.

10. The radius OA of the circle BAC is produced to D, and in OA a point P is taken such that OA is a mean proportional between OP and OD.

Prove that if BPC be any chord passing through P, the angles BDP and CDP will be equal.

11. If the vertical angle of a triangle be bisected by a straight line which likewise cuts the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained by the segments of the base, together with the square on the straight line which bisects the angle.

ALGEBRA.

1. If a, b, c, and d be four numbers such that a is greater than b, and c greater than d, prove that

$$(a-b)(c-d)=ac-ad-bc+bd.$$

Assuming the universality of this equation, deduce the rule of signs in multiplication.

2. Multiply

$$x^6-x^5+5x^3+16x^2+26 \text{ by } x^3+x^2-11x-12;$$

and resolve into factors the expressions

$$(a+4b+5c)^2-(2a-b-2c)^2, \text{ and } x^2-19x+84.$$

3. Prove the relations:

(α) $\{(ax+by)^2+(ax-by)^2\}\,\{(ay+bx)^2+(ay-bx)^2\}$

$$= 4x^2y^2\,(a^2 \pm b^2)^2 + 4a^2b^2\,(x^2 \mp y^2)^2.$$

(β) $x^3\,(y-z)+y^3\,(z-x)+z^3\,(x-y)$

$$= (z-y)\,(x-z)\,(y-x)\,(x+y+z).$$

4. Find the highest common divisor, and the least common multiple of the expressions

$$x^4 - 795x^2 + 529, \text{ and } x^4 - 845x^2 + 1450x - 621.$$

5. If $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$, prove that each of these fractions

$$= \frac{x+y+z}{a+b+c} = \sqrt{\left(\frac{x^2+y^2+z^2}{a^2+b^2+c^2}\right)}.$$

Having given $\dfrac{y+z}{b+c} = \dfrac{z+x}{c+a} = \dfrac{x+y}{a+b}$,

prove that $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$.

6. Solve the equations:

(α) $(x+1)\,(x+4)+(x+2)\,(x+3)$

$$= 2\,(x+5)\,(x+6) - 122.$$

(β) $8x^2 - 70x + 143 = 0.$

(γ) $(x^3 - 5x^2 + 15x + 198)^{\frac{1}{3}} - x = 4.$

(δ) $\left.\begin{aligned} x^2 + 4y^2 &= 100 \\ x + 2y &= 14 \end{aligned}\right\}.$

7. Two trains, the lengths of which are respectively 152 feet and 112 feet, are travelling with uniform velocities on parallel rails in opposite directions, and are observed to pass each other in six seconds; but, when they are moving in the same direction, with the same velocities as before, the faster train is observed to pass the other in twelve seconds; find, in miles per hour, the velocities of the trains.

8. Having given the first term and the common ratio of a geometrical progression, find the sum of the first n terms, and find the limit of this sum when n is indefinitely increased, assuming that the common ratio is less than unity.

Prove that the product of the two series

$$1 + r + r^2 + r^3 + \dots \text{ to infinity,}$$

and $$1 - r + r^2 - r^3 + \dots \dots\dots\dots,$$

is equal to $$1 + r^2 + r^4 + r^6 + \dots \dots\dots\dots,$$

r being less than unity.

9. If $a + \sqrt{(b)} = x + \sqrt{(y)}$, where a and x are rational quantities and $\sqrt{(b)}$ and $\sqrt{(y)}$ irreducible surds, prove that $a = x$ and $b = y$.

Simplify the expression

$$(5^{\frac{3}{4}} + 5^{\frac{1}{2}}.3^{\frac{1}{4}} + 5^{\frac{1}{4}}.3^{\frac{1}{2}} + 3^{\frac{3}{4}}) \times (5^{\frac{1}{4}} - 3^{\frac{1}{4}}),$$

and find the square root of

$$16 - 8\sqrt{(3)}.$$

10. Find an expression for the number of permutations of n things taken r together, and deduce an expression for the number of combinations of n things taken r together. Find n when the number of combinations of $2n$ things taken 3 together is to the number of combinations of n things taken 4 together in the ratio of 44 to 3.

11. If α and β be the roots of the equation

$$x^2 - px + q = 0,$$

prove that $$\alpha + \beta = p, \text{ and } \alpha\beta = q.$$

Find the condition that the preceding equation may have its roots equal, and find the value of m when the equation

$$2mx^2 + (8m + 4)x + 12m + 1 = 0$$

has its two roots equal.

12. Enunciate the Binomial Theorem, and prove it for a positive integral index.

Expand $(1-x)^{-3}$, and $(1-2x)^{-\frac{1}{2}}$, each to four terms, and in each case write down the coefficient of x^r.

PURE MATHEMATICS.

Logarithm books will not be required.

1. State the rule for finding the convergents of a continued fraction, and find the 5th convergent to $\sqrt{(27)}$.

Find also the value of

$$\cfrac{2}{1-\cfrac{2}{9-\cfrac{2}{1-\&c.}}}$$

to infinity.

2. If $a^x = 1 + a_1x + a_2x^2 + \&c.$ ad infinitum, find a_1 in terms of a.

Define e, and deduce the expansion of $\log_e(1+x)$.

3. Prove that

$$\log a + \log b = \log ab.$$

Hence shew that

$$\log 2 + \log\left(1 - \frac{1}{2^2}\right) + \log\left(1 - \frac{1}{3^2}\right) + \&c. \text{ to } n \text{ terms}$$
$$= \log(n+2) - \log(n+1).$$

4. Three people at a railway station going to different places call for their tickets at the same instant and take them up at random. Find the chance (1) that each one of the three gets his right ticket, (2) that each one of them gets a wrong ticket.

5. State the principle of indeterminate coefficients.

Find three simple fractions equivalent to

$$\frac{2b^2}{(x-a+b)(x-a)(x-a-b)}.$$

6. Prove that

$$\sin(A+B)=\sin A\ \cos B+\cos A\ \sin B.$$

And assuming that this is true for all values of A and B, deduce the corresponding formula for $\cos(A+B)$.

7. In a triangle given $A=42°$, $b=1706$, $a=1401$: find B and C, having given

$$L\ \sin 42° = 9{\cdot}8255109,\ \log 17{\cdot}06 = 1{\cdot}2319790,$$
$$\log 14{\cdot}01 = 1{\cdot}1464381,\ L\ \sin 54°\ 34' = 9{\cdot}9110460,$$
$$L\ \sin 54°\ 35' = 9{\cdot}9111359.$$

8. Prove that in any triangle ABC whose sides are a, b, and c,

$$\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2}.$$

If $b=\sqrt{(3)}+1$, $c=2$ and $A=60°$, solve the triangle.

9. Prove that if a be a root of the equation $f(x)=0$, then $f(x)$ will be divisible by $(x-a)$.

10. Shew how to transform a given equation $f(x)=0$ into another whose roots are, respectively, the negatives of the roots of the given equation. Apply this or any other method to the solution of the equation

$$x^4+x^3+x^2+2x-2,$$

it being known that there are two roots of the form $+a$ and $-a$.

11. Find the equation of the straight line in the form

$$y=mx+c,$$

interpreting the constants.

Find the equation of the straight line which joins the point $(6,-2)$ with the point of intersection of the straight lines

$$6y-3x+10=0,$$
$$6x+3y-10=0.$$

12. Assuming the equation of the tangent to the parabola at the point x', y', find the equation of the normal to the curve at the same point.

If the normal at L, one extremity of the latus rectum, meet the axis in M and the parabola again in P, and the normal at P meet the axis in N, prove that MN is equal to the latus rectum.

13. Find the locus of the middle points of a system of parallel chords in an ellipse.

Additional Questions.

A. Find the general term in the expansion of $(a_0 + a_1x + a_2x^2 + \&c.)^n$.

Find the coefficient of x^{12} in the expansion of

$$(1 - 3x^2 + 4x^3 + x^5)^{20}.$$

B. State and prove the exponential expressions for $\cos\theta$ and $\sin\theta$.

Apply to find x and y in terms of a and b from the equation

$$\log_e\{a + b\sqrt{(-1)}\} = x + y\sqrt{(-1)},$$

where x, y, a, and b are all real.

C. ABC and DEF are two triangles whose sides are a, b, c, and d, e, f. Prove that if $d = a\cos A$, $e = b\cos B$, and $f = c\cos C$, then the area of DEF is to the area of ABC as $\cos A \cos B \cos C$ is to 1.

D. Prove that the equation

$$A(x^2 - y^2) + 2Hxy + C = 0,$$

represents an equilateral hyperbola if the axes are rectangular.

MIXED MATHEMATICS.

1. Assuming the parallelogram of forces, prove the propositions called the triangle of forces, and the polygon of forces.

Enunciate the converses of these propositions, and state whether they are necessarily true.

2. Investigate the conditions of equilibrium of a system of forces acting upon a rigid body in one plane.

Four rods freely jointed together form a parallelogram *ABCD*, and are fixed in a vertical plane by pegs at *A* and *C* in the same vertical line; shew that the strains at *B* and *D* act along *BD*.

3. If a heavy body be suspended from a fixed point about which it can swing freely, prove that it will rest with its centre of gravity in the vertical line through the fixed point.

A heavy uniform rod, 15 inches in length, is suspended from a fixed point by two strings fastened to its ends, the lengths of the strings being respectively 9 inches and 12 inches.

If θ be the angle at which the rod is inclined to the vertical, prove that $25 \sin\theta = 24$.

4. Find the ratio of the power to the weight in the system of pulleys in which each hangs by a separate string, neglecting the weights of the pulleys.

Prove for this case the truth of the principle of virtual velocities.

5. Explain the meaning of the equation

$$P = Mf.$$

A weight of 50 lbs. is moved in a straight line along a smooth horizontal plane by a constant force, which would support statically a weight of 5 lbs.; supposing the weight initially at rest, find its velocity at the end of three seconds.

6. A heavy body falls freely; what is the numerical measure of its acceleration—

1st. When a foot and a second are units of length and time;

2nd. When a yard and a minute are the units?

A body is projected vertically upwards with a velocity of 50 feet per second; find the greatest height it attains, and the two times at which the body is at half its greatest height.

7. A heavy particle is projected along the surface of a smooth inclined plane; prove that its path is a parabola.

8. A weight of 50 lbs. on a smooth horizontal table is connected by a string, passing over the edge of the table, and perpendicular to the edge, with a weight of 5 lbs.; the system having no initial velocity, find its velocity at the end of three seconds. Find also the tension of the string.

9. Two imperfectly elastic balls, moving with given velocities, impinge directly on each other; find the velocity of each after impact.

What is the condition that they may interchange velocities?

10. Explain how fluid pressure at a point is measured. Distinguish between whole pressure on a surface and resultant pressure.

11. Prove that the pressure of a liquid at rest is the same at all points of the same horizontal plane.

The weight of a cubic foot of water being 1,000 oz., what is the depth of a lake when the pressure on a square yard at the bottom of it is equal to $56\frac{1}{4}$ tons?

12. Describe the Common Hydrometer, and its use in comparing the specific gravities of two liquids.

PURE MATHEMATICS.

1. Prove that the three angles of a spherical triangle are together greater than two right angles, and less than six right angles.

2. If ABC be a spherical triangle, prove that

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$$

If $b + c = \pi$, prove that $\sin 2B + \sin 2C = 0$.

3. Find expressions for the angular radii of the small circles inscribed in, and circumscribed about, a spherical triangle.

If r and R be these radii, prove that in the case of an equilateral triangle

$$\tan R = 2 \tan r.$$

4. Differentiate with regard to x the following expressions:

$$(2x-a)(3x-a)(4x-a), \ (\sin x)^{-1}, \ \sin^{-1} x,$$

$$\log \frac{x + \sqrt{(a^2 - x^2)}}{x - \sqrt{(a^2 - x^2)}}, \text{ and } \log \{\tan (\log x)\}.$$

5. State and prove Leibnitz's theorem for the differentiation of the product of two functions.

Find the n^{th} differential coefficient of the expression

$$x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y.$$

6. If $y = \varepsilon^{ax} \sin bx$, prove that

$$\frac{d^r y}{dx^r} = (a^2 + b^2)^{\frac{1}{2}r} \varepsilon^{ax} \sin (bx + r\phi),$$

where $$\tan \phi = \frac{b}{a}.$$

Employ Maclaurin's theorem to expand $\epsilon^{ax} \sin bx$ in a series of powers of x.

7. Prove that when a function has a maximum or minimum value its differential coefficient either vanishes or is infinite. Is the converse of this proposition true?

Determine, and distinguish between, the maximum and minimum values of the expression

$$2x^3 - 15x^2 + 36x - 23.$$

8. If ϕ be the angle between the tangent and radius vector at any point (r, θ) of a curve, prove that

$$\cos\phi = \frac{dr}{ds}, \text{ and } \sin\phi = \frac{rd\theta}{ds}.$$

Prove that, in the case of the curve,

$$r^2 = a^2 \cos 2\theta, \quad \phi = \tfrac{1}{2}\pi + 2\theta.$$

9. Find the asymptotes of the following curves:

(α) $(x-1)y^2 = x^2 - 2.$

(β) $r(\theta - \alpha) = a(\theta - 2\alpha).$

10. Shew how to find the envelope of the system of curves represented by the equation $f(x, y, \alpha) = 0$, α being a variable parameter.

Find the envelope of the lines

$$\frac{x}{a}\cos\alpha + \frac{y}{b}\sin\alpha = 1.$$

11. Integrate with regard to x the expressions

$$\frac{\cos x}{\sin^2 x}, \quad \frac{1}{x^2 - a^2}, \text{ and } \frac{1}{(x-a)(x-2a)}.$$

12. Shew how to find the area of a curve in polar coordinates.

Find the area of the curve $r^2 = a^2 \cos 2\theta$.

13. A plane cuts the axes of reference in the points A, B, C, at the distances a, b, and c from the origin.

Prove that its equation is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

If G be the centre of gravity of the area of the triangle ABC, and D the foot of the perpendicular from the origin (O) on the plane, find the coordinates of G and D, and the cosine of the angle GOD, the axes being rectangular.

14. Find the areas of the three circles in which the sphere

$$(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2=c^2$$

is intersected by the coordinate planes.

If this sphere, remaining unchanged in size, move about so that its centre is always at a constant distance from the origin, prove that the sum of the areas of the three circles of intersection will be constant.

ANSWERS AND HINTS FOR SOLUTION.

DIRECT, MAY, 1873. PP. 1–7.

Euclid, Bk. I.

5. Bisect the given line AB at C, through which draw DCE perpendicular to AB; make DC, CE each equal to AC or CB, and join AD, DB, BE, EA. 6. Use Euc. I. 20.

Arithmetic.

1. $3\frac{31}{120}$. 2. $17\frac{7}{8}$. 3. $\frac{4}{7}$.
4. $2\frac{10}{17}$. 5. 296·09218. 6. 9·8202.
7. ·3421575. 8. 148·279. 9. £2. 1*s.* 8*d.*
10. 149328 oz. 11. £7. 8*s.* 6½*d.* 12. £155. 10*s.* $4\frac{4}{5}$*d.*
13. $9\frac{5}{24}$. 14. $4\frac{343}{380}$. 15. $2\frac{2}{3}$.
16. $\frac{41}{58}$. 17. 3795 min.
18. 3 tons 8 cwt. 1 qr. 14 lbs. 8·96 oz. 19. 36·1875.
20. ·0196. 21. 74·8.
22. 120 sq. yds. $8\frac{2}{9}$ sq. ft. 23. £1449.
24. 2½ per cent. 25. £113. 4*s.* 7½*d.* 26. 4 horses.

Algebra, Logarithms, and Mensuration.

1. $\frac{1}{4}$, 4. 2. (A) $2(2b-3c)$; (B) $a^6-a^4b^2+a^2b^4-b^6$; (C) $4x^2-12ax+9a^2$; (D) $a^2+ab\sqrt{3}+b^2$; (E) $1-2x^{\frac{1}{2}}+3x$.

3. $(1-x)(1-x^2)^2$; $\dfrac{3(x^2-7ax+12a^2)}{2(x^2+7ax+12a^2)}$.

4. $\dfrac{1}{\sqrt[3]{3}}$; express the different terms as powers of 2 and 3 with fractional indices, and use *For.** p. 35, ll. 9, 10.

5. (A) 15; (B) $x=16$, $y=7\frac{1}{5}$, $z=12\frac{4}{5}$.

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* Arnett's *Mathematical Formulæ* (Tomlin, Cambridge; Simpkin, Marshall and Co., London).

6. 4 mls. per hour; 8 mls. per hour.

7. (A) ± 7; (B) 5, $\frac{7}{31}$; (C) 2, 6, − 4. The left-hand side is $x^3 - 2^3$; therefore $x - 2$ divides out. See *For.* p. 40, l. 1.

8. 598 sq. ft. 10′8″ = 598 sq. ft. 128 sq. in.

9. See *For.* pp. 10, 51; 3, 9, 27, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$; √(10); 8286·97. 10. 3 rd. 21 pl. 1 sq. yd. 3 sq. ft. 108 sq. in.

11. 2400 √(3) c. ft. 12. 3600; 60 √(3).

Geometry and Trigonometry.

2. Shew that the bisector of the angle bisects at right angles the straight line joining the extremities of the two equal straight lines. 4. The angle at the centre of the circle and the angle opposite to it in the original triangle are equal to two right angles; the result follows by Euc. III. 20. 5. The sides of the inscribed square respectively bisect the four squares, which make up the described square. 8. Use Euc. VI. C.

9. See *For.* pp. 75, 76; 120°, 108°.

10. $\frac{180^\circ + \pi^\circ}{2\pi}$, $\frac{180^\circ - \pi^\circ}{2\pi}$. 11. *For.* p. 77.

14. *For.* p. 80, l. 2; we have

$$\frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}} = \frac{2ab}{a^2+b^2};$$

therefore
$$\frac{1+2\tan\frac{A}{2}+\tan^2\frac{A}{2}}{1-2\tan\frac{A}{2}+\tan^2\frac{A}{2}} = \frac{a^2+2ab+b^2}{a^2-2ab+b^2}$$

by *For.* p. 38, l. 6; therefore

$$\frac{1+\tan\frac{A}{2}}{1-\tan\frac{A}{2}} = \pm\frac{a+b}{a-b};$$

therefore
$$\frac{2\tan\frac{A}{2}}{2} = \frac{2b}{2a} \text{ or } \frac{a}{b}.$$

16. Find C, and use *For.* p. 84, l. 5; $b = 89{\cdot}646$.

17. *For.* p. 84, l. 11; also, $\cot\frac{A}{2} = \tan\left(90^\circ - \frac{A}{2}\right) = \tan 61^\circ 46'$.
See also *For.* p. 83; 74° 32′ 44″, 48° 59′ 16″.

18. Produce AD, making $DE = AD$; then BE is parallel to AC.

$$\frac{\sin BAD}{BE} = \frac{\sin ABE}{AE} = \frac{\sin A}{AE}, \text{ since } \angle ABE = 180^\circ - A;$$

$$\text{but} \quad AE^2 = b^2 + c^2 - 2bc \cos ABE$$
$$= b^2 + c^2 + 2bc \cos A;$$

$$\text{therefore} \quad \sin BAD = \frac{b \sin A}{\sqrt{(b^2 + c^2 + 2bc \cos A)}}.$$

DIRECT, AUGUST, 1873. PP. 8–14.

Euclid, Book I.

5. Draw a diagonal, and use Euc. I. 8.
6. Draw the diagonal through A, and use Euc. I. 19.

Arithmetic.

1. $3\frac{17}{90}$. 2. $6\frac{17}{18}$. 3. $\frac{3}{8}$.
4. $3\frac{7}{16}$. 5. 73·547045. 6. 292·1468.
7. ·289918. 8. 34·114. 9. £10. 5*s.*
10. 2 wks. 11 hrs. 26 min. 24 sec. 11. 19*s.* $11\frac{1}{4}$*d.*
12. £190. 6*s.* $4\frac{4}{5}$*d.* 13. $7\frac{9}{100}$. 14. $4\frac{15}{16}$.
15. $8\frac{7}{16}$. 16. $\frac{15}{22}$. 17. 90·16 yds.
18. 82·94 hrs. 19. ·319792. 20. ·0242.
21. $\cdot 1\dot{6}$. 22. 174 c. yds. 22 c. ft. 544 c. in.
23. £5062. 10*s.* 24. 13 yrs. 25. £1621. 9*s.* 10*d.*
26. £30.

Algebra, Logarithms, and Mensuration.

1. $x^5 - 8x^4 + 29x^3 - 79x^2 + 108x - 45$; 10611;
(α) $9(x+a)(x-a)$; (β) $(x-a)(x-b)$; (γ) $(x-11)(x-7)$.
2. (α) $x^3 + 3x^2 + 15x - 11$; (β) $x^2 + y^2 + z^2 - xy - yz - zx$;
(γ) $x^2 - 3xy + y^2$.
3. $x - 3$; $x^5 - 10x^4 + 75x^3 - 309x^2 + 882x - 1323$; 4 or 5.
4. Add and subtract 1, and divide; see *For.* *p.* 37.
5. $98 - 8\sqrt{(15)} + 6\sqrt{(5)} - 40\sqrt{(3)}$; $5\sqrt{(5)} - 3\sqrt{(3)}$; 16·4248.
6. Take $a - 1$, a, $a + 1$, as the three numbers.

7. (α) 11; (β) 4; (γ) 49, 39; (δ) $\pm\sqrt{(21)}$, the roots $\pm\sqrt{(77)}$ belong to the equation, $x^2-4\sqrt{(x^2+4)}=41$; (ε) $x=\pm 11, \pm 10$; $y=\pm 10, \pm 11$.
8. 87, see *Form. p.* 40, *l.* 5 *from bottom.*   9. 60.
10. 4, 12, 4, – 12; 2·6883872; 1·118803.
11. 595 ft.; 84966 sq. ft.   12. 11·51388; 8·594208.
13. $42522\frac{9}{28}$ tons.   14. 186 c. yds. 4·56 c. ft.; 279 sq. yds. 2·28 sq. ft.; 162 c. ft. 24·24 c. ft.

Geometry and Trigonometry.

2. Use Euc. II. 7.
3. Let AD, BC intersect in E;

$$\angle DAB = \angle DCB = \angle ABC; \text{ therefore, } EA = EB;$$

similarly $EC = ED$; thus AC, BD are equal, and therefore equally distant from the centre.
4. Let the right angle be at C, and let D, E, F be the points of contact of the inscribed circle in the sides BC, CA, AB respectively. Then, if O be the centre of this circle, $CEOD$ is a square; and, therefore CE, CD together = the diameter of the inscribed circle. Also by *For. p.* 94, AE, and BD together $= AB$, the diameter of the circumscribing circle.
8. Todhunter's *Euclid*, Appendix, Art. 55.
9. *For. p.* 75; $\dfrac{270^\circ}{180-\pi}$.   10. 45°.
13. Use *For. p.* 28, *l.* 1, and Euc. VI., C.
14. Use $\sin A = 2\sin\dfrac{A}{2}\cos\dfrac{A}{2}$.   15. 41° 41′ 28″.
16. 100° 47′ 0″·9; 14° 12′ 59″·1.   17. Use *For. p.* 80, *l.* 6.

WOOLWICH, MAY, 1873. PP. 15–28.

Arithmetic and Logarithms.

1. $1\frac{115}{252}$.   2. $12\frac{7}{15}$.   3. $3\frac{16}{21}$.
4. $\frac{39}{50}$.   5. 773·65738.   6. 618·5246.
7. ·062068.   8. ·0000589.   9. ·067.
10. $7\frac{386}{1725}$.   11. $4\frac{28}{33}$.   12. $1\frac{3}{11}$.
13. $\frac{41}{111}$.   14. 8929·59 min.   15. 4994 yds.
1 6.·056536.   17. 168·960.   18. ·04875.

19. 7427 ft. 20. 1*s.* 10½*d.* 21. £5946. 15*s.* 3¾*d.*
22. £594. 23. 152550 sq. ft. 24. 52½ days.
25. £752. 6*s.* 10½*d.* 26. £7524. 11*s.* 3*d.* 27. 131·2425.
28. ·69897, ·7781513, $\bar{2}$·3344539, $\bar{1}$·9148063.
29. 62573; $9\frac{3}{23}$. 30. £2202. 10*s.* 31. £993. 4*s.* 6*d.*

Geometry.

4. Let *O* be the common centre, and *ACDB* the straight line intersecting the outer circle in *A*, *B*, the inner circle in *C*, *D*. Draw *OE* perpendicular to *AB*. Then *AE* = *EB*, and *CE* = *ED*; therefore, *AC* = *DB*.
6. (i) Let *A* be the given point, and *BC* the given straight line; draw *AD* perpendicular to *BC*, and, if necessary, produce *DA* to *E*, making *DE* equal to the given radius. Through *E* draw *EF* parallel to *BC*, and from the centre *A* at the distance *DE* describe a circle cutting *EF* at *F*. Then the circle with *F* as centre and radius *FA* will be the circle required. Since the circle described from the centre *A* will cut *FE* in two points, there are two solutions. (ii) Let *A* be the centre of the given circle, and *BC* the given straight line. Draw a straight line *DE* parallel to *BC* at a distance from it equal to the given radius, and on the same side of it as *A*. Describe a circle with *A* as centre and radius equal to the sum of the radii of the required circle and the circle whose centre is *A*. Let this circle cut *DE* at *E*. Then the circle with the given radius and with centre *E* will touch *BC* and the given circle. Since the circle described from the centre *A* and radius *AE* will meet *DE* in two points, there are two solutions.
7. Let the common chord *AB* produced cut the common tangent *CD* in *E*. Then the squares on *EC*, *ED* are each equal to the rect. *EB*, *EA*.

Algebra.

1. 1. 2. $\frac{2y^3}{x^2-y^2}$; $a^4+2a^2b^2+b^4$.

3. $x^{\frac{4}{3}}-a^{\frac{1}{3}}x-ax^{\frac{1}{3}}+a^{\frac{4}{3}}$; $\frac{cx-d}{ax+b}$.

4. (1) Use *For.* p. 36, l. 7; 2. (2) $x=4$, $y=3$.
(3) ± 3. (4) $\begin{array}{l|c|c|c|}x=4 & -\frac{2}{3} & \frac{2}{3} & -4 \\ y=5 & 2\frac{2}{3} & 8\frac{1}{3} & 6 \\ z=6 & 8\frac{1}{3} & 2\frac{2}{3} & 5\end{array}$.

5. 200 ac. arable at £40 per acre; 180 ac. pasture at £50 per acre.

6. See *For.* p. 42. If Δ be the area of a triangle, a the length of the base, p the perpendicular, we have $p=\frac{2\Delta}{a}$; therefore p varies directly as Δ and inversely as a. $x^3-xy+6=0$.

7. $\dfrac{\left(1-\dfrac{1}{4a}\right)^3}{1-\left(1-\dfrac{1}{4a}\right)^3}$, $\frac{3781}{5184}$, $8\frac{1403}{5184}$.

8. $\dfrac{(n+1)\,a}{2n}$; 7, 9, 11...23.

9. $1-x+\dfrac{2x^2}{9}$. We have $a+c>2\sqrt{(ac)}$, and $a+c=\dfrac{2ac}{b}$; therefore $\dfrac{2ac}{b}>2\sqrt{(ac)}$; therefore $\sqrt{(ac)}>b$; therefore $a^{\frac{3}{2}}c^{\frac{3}{2}}>b^3$; but $a^3+c^3>2a^{\frac{3}{2}}c^{\frac{3}{2}}$; *a fortiori*, $a^3+c^3>2b^3$. 10. See *For.* pp. 45, 46.

11. $(-1)^r\dfrac{n(n+1)\ldots(n+r-1)}{\underline{\lfloor r}}x^r$; the fifth term.

Pure Mathematics.

1. (i) Write $x+y+z=a$; then

$$(y+z-x)^2+(z+x-y)^2+(x+y-z)^2=(a-2x)^2+(a-2y)^2+(a-2z)^2$$
$$=3a^2-4a(x+y+z)+4(x^2+y^2+z^2)=4(x^2+y^2+z^2)-a^2$$
$$=4(x^2+y^2+z^2)-(x+y+z)^2.$$

(ii) Expand, and then add and subtract $2abcd$.
(iii) In four ways.

2. $8\frac{1}{2}$, $6\frac{1}{3}$; see *For.* p. 68; $\dfrac{b^2-2ac}{a^2}$; $\dfrac{3abc-b^3}{a^3}$, use *For.* p. 36, l. 5. 3. n^2; $\frac{1}{2}(3^n-1)$; $\frac{1}{6}n(n+1)(2n+1)$.

4. 4; $r+1$; $\dfrac{1.3.5}{2^3.\underline{|3}}$; $\dfrac{1.3.5\ldots(2r-1)}{2^r.\underline{|r}}$; ${}_nC_r + {}_nC_{r-1}$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{1.2.3\ldots r} + \frac{n(n-1)(n-2)\ldots(n-r+2)}{1.2.3\ldots(r-1)}$$

$$= \frac{n(n-1)\ldots(n-r+2)}{\underline{|r-1}}\left(\frac{n-r+1}{r}+1\right)$$

$$= \frac{(n+1)(n)(n-1)\ldots(n-r+2)}{\underline{|r}} = {}_{n+1}C_r.$$

5. $1+\dfrac{1}{1+}\dfrac{1}{2+}\dfrac{1}{7+}\dfrac{1}{1+}\dfrac{1}{1+}\dfrac{1}{2}$; $1, 2, \frac{5}{3}, \frac{37}{22}$;

$3+\dfrac{1}{6+}\dfrac{1}{6\ldots}$; $3, \frac{19}{6}, \frac{117}{37}, \frac{721}{228}$.

6. $\dfrac{\sqrt{(3)}-1}{2\sqrt{(2)}}$, $\dfrac{\sqrt{(3)}+1}{2\sqrt{(2)}}$. 7. 90°, 30°, 60°.

8. The given values of a and A are common to the two triangles, so that the result follows at once from the value of R. 9. *For.* p. 10; 6, -7.

10. *For.*p.47; $\frac{30}{91}$. 11. No. 12. $x+y-3=0$.

13. $p = \dfrac{y'^2-2ax'}{\sqrt{(y'^2+4a^2)}} = \dfrac{2ax'}{2\sqrt{(a)}\sqrt{(a+x')}}$.

14. In the ellipse, $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, the distance of either focus from the origin $=\sqrt{(a^2-b^2)}$. $\pm\sqrt{\left(\dfrac{8}{3}\right)}$, $\pm\dfrac{1}{\sqrt{(3)}}$. Fig. 1.

Mechanics and Hydrostatics.

1. Todhunter's *Mechanics for Beginners*, Art. 109.
2. Let AB be the diameter, and C any position of the particle. The attracting forces may be represented by the chords CA, CB, and by the parallelogram of forces their resultant is represented by the diameter through C.

4. $\left\{2+\dfrac{3}{\sqrt{(2)}}\right\}$ ft., $\{4+3\sqrt{(2)}\}$ ft.

5. (i) The curved surface of a cone is the ultimate form of the surface of a pyramid formed by triangles, having the vertex of the cone for their common vertex, and having for their bases the sides of a polygon inscribed in the circle, which is the base of the cone. Now the centre of gravity of each triangle is on the line joining the vertex to the middle point of its base, at $\frac{2}{3}$rds. of the length of this line from the vertex; thus the centre of gravity of the surface is on the axis of the cone at $\frac{2}{3}$rds. of its length from the vertex. (ii) The centre of gravity of each triangle coincides with the centre of gravity of 3 equal particles placed at its angular points, and therefore with the centre of gravity of 6 equal particles placed at the angular points of the hexagon.

6. $W(\sin\alpha + \mu \cos\alpha)$, $W(\sin\alpha - \mu \cos\alpha)$.

7. 38400. 8. $\tan^{-1}\frac{12}{11}$. 9. $\frac{\pi v^4}{g^2}$; $\frac{v^2}{g(1+\sin\alpha)}$; $\frac{v^2 \sin\alpha}{g \cos^2\alpha}$.

11. $\tan^{-1}\frac{3w}{(5a)^3}$, where w is the weight of the lid in lbs., and a the length of an edge of the box.

12. $\frac{1}{2}\{a + l + h + \sqrt{(a^2 + l^2 + h^2 - 2al + 2ah + 2lh)}\}$, where l is the length of the tube, and a the depth to which the open end is immersed. *A.* Todhunter's *Anal. Stat.*, Chap. VII. Take a corner of the cube as origin, and let P act along the axis of x, Q parallel to the axis of y, R parallel to that of z. The equations of the central axis are

$$\frac{(R-Q)a - yR + zQ}{P} = \frac{-Ra - zP + xR}{Q}$$

$$= \frac{-xQ + yP}{R} = \frac{\{(R-Q)P - QR\}a}{\sqrt{(P^2 + Q^2 + R^2)}}.$$

B. Let P be the attached weight, θ the inclination of the string to the horizontal line in the plane, ϕ the inclination of the direction in which the weight will begin to move. Then the forces perpendicular to the plane are $W\cos\alpha$ and R; thus

$$W\cos\alpha = R.$$

The forces in the plane are P, μR, $W\sin\alpha$; therefore, resolving in the plane along the horizontal line and at right angles to it,

$$P\cos\theta = \mu R\cos\phi,$$

$$P\sin\theta - W\sin\alpha = \mu R\sin\phi.$$

C. Todhunter's *Anal. Stat.* Cap. XIII. *D.* Tait and Steele's *Part. Dyn.* Cap. V.

Pure Mathematics.

4. (1) $\dfrac{3a\,(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{5}{2}}}$;

(2) $(\log x)^{x-1} + (\log x)^{x}\log(\log x)$; (3) $-\dfrac{\sin(\tan^{-1}x)}{1+x^2}$.

5. $e^{mx}\left\{m^r x^n + nrm^{r-1}x^{n-1} + n(n-1)\dfrac{r(r-1)}{\lfloor\underline{2}}m^{r-2}x^{n-2} + \dots\right.$

$$\left.+\frac{\lfloor\underline{n}}{\lfloor\underline{n-r}}x^{n-r}\right\}.$$

6. $e\left(1+x+x^2+\dfrac{5x^3}{6}\right)$; $-2 \pm \sqrt{(6)}\sqrt{(x)} \mp \dfrac{x^{\frac{3}{2}}}{2\sqrt{(6)}} \mp \dfrac{x^{\frac{5}{2}}}{48\sqrt{(6)}}$.

7. A minimum when $x = 0$, neither when $x = -1$; if a^3 be the given volume, r the radius of the base, h, h' the heights of the cones, $h = h' = r\sqrt{(2)} = a\sqrt[3]{\left(\dfrac{3}{\pi}\right)}$.

8. $Y - y = \frac{1}{2}\left(\varepsilon^{\frac{x}{a}} - \varepsilon^{-\frac{x}{a}}\right)(X - x)$. 9. $\rho = \dfrac{y^2}{2a}$.

11. (2) $\dfrac{1}{1+\tan x} = \dfrac{1}{2} + \dfrac{1}{2}\cdot\dfrac{\cos x - \sin x}{\sin x + \cos x}$; thus

$$\int\frac{dx}{1+\tan x} = \frac{x}{2} + \log\sqrt{(\sin x + \cos x)}.$$

$$\int(a^2+x^2)^{\frac{n}{2}}dx = \frac{x\,(a^2+x^2)^{\frac{n}{2}}}{n+1} + \frac{na^2}{n+1}\int(a^2+x^2)^{\frac{n}{2}-1}dx.$$

12. $\frac{24}{25}$.

13. (i) A right cone, vertex (a, b, c), axis parallel to (l, m, n), cos (semi-vertical angle) $= \dfrac{1}{\kappa}$. (2) Distance of point on the locus from (a, b, c) = projection of

distance from origin on (l, m, n). By subtraction the surfaces intersect in the two planes

$$lx + my + nz = \pm \kappa \{(x-a)\,l + (y-b)\,m + (z-c)\,n\}.$$

C. Figures 2 and 3.

WOOLWICH, SEPTEMBER, 1873. PP. 28–41.

Arithmetic and Logarithms.

1. $4\frac{101}{120}$.
2. $1\frac{5}{9}$.
3. $10\frac{1}{8}$.
4. $22\frac{5}{13}$.
5. 1652·64895.
6. 21·9515.
7. ·1619084.
8. ·3386.
9. ·25.
10. $8\frac{37}{70}$.
11. $1\frac{17}{22}$.
12. $20\frac{13}{16}$.
13. $\frac{71}{120}$.
14. 22748·19 min.
15. 419·1 yds.
16. 1975·06.
17. 126·042.
18. $\cdot 0210\dot{6}$.
19. 337734 in.
20. £159. 2*s.*
21. £5390. 13*s.* $1\frac{1}{2}$*d.*
22. £1081.
23. 250695 sq. ft.
24. 65 ac.
25. £1443. 2*s.* $3\frac{1}{2}$*d.*
26. £980. 3*s.* 7*d.*
27. 41·3394.
28. $\bar{1}\cdot 7499591$; $\bar{2}\cdot 2108534$; 10.
29. 15975; 6·12.
30. £7068. 15*s.*

Geometry.

1. If *AE*, *BE* be the bisectors, the angle *AEB* together with half the sum of the angles *A* and *B* = two right angles, and half the sum of the angles *A* and *B* together with half the sum of the two remaining angles = two right angles.
3. Let the straight line *AB* be divided at *H* so that the rect. *AB*, *BH* = sq. on *AH*. Take *AK* = *HB*; therefore *AH* = *KB*. Then the rect. *AH*, *HK* together with the rect. *AH*, *AK* = sq. on *AH* (Euc. II. 2) = rect. *AB*, *AK* (Euc. II. 11) = sq. on *AK* together with the rect. *AK*, *KB* (Euc. II. 3). Therefore the rect. *AH*, *HK* = sq. on *AK*.
5. All the triangles on the base *AB*, having the same vertical angle, have their vertices on the circumference of a segment, of which *AB* is the base. Let *ACB* be the isosceles triangle, and *CE* the tangent at *C*; *CE* is parallel to *AB*. Let *ADB* be any other of the triangles; produce *AD* to

meet CE at E, and join BE. Then the triangle $ACB = AEB > ADB$.

6. See *For.* p. 16, Prop. V. Circles will go round $BFEC$, $FDBP$, and $PDCE$; thus the angle EDA = the angle FCE = the angle FBE = the angle FDA.
8. See question 4 on p. 13.
10. Let ACB be the given arc; divide AB at D so that AD is to DB in the given ratio, and produce AB to E so that AE is to EB in the given ratio. On DE describe the semicircle DCE cutting the given arc at C, the point required.
11. With the figure of Todhunter's *Euclid*, VI. C., we have rect. AD, EA = rect. BA, AC; therefore

$$EA : 1 :: BA.AC : AD;$$

therefore $\quad EA : BC :: BA.AC : AD.BC.$

Algebra.

1. 26; 0; -4.
2. (1) $x^4 - 2(a^2+b^2)x^2 + (a^2-b^2)^2$; (2) $x^2 - 2xy + y^2$.
3. (1) Take $2n(2n-1)$ out of the two first terms, and $2n(2n+1)$ out of the two last; (2) use *For.* p. 35, ll. 9, 10. 4. $\sqrt{(5)} - \sqrt{(2)}$; 1.
5. (1) 5, 7; (2) 4, $-4\frac{1}{5}$; (3) $3\frac{1}{2}$, 10, -3. Use *For.* p. 35, l. 9.
6. Blue, 4*s.* 6*d*; scarlet, 7*s.* 6*d.*
7. $y^3 + y^2(18-p^2) + y(81-2pr) - r^2 = 0$.
8. $a + (2n-5)b$; $n\{a + b(n-4)\}$; 6, -9.
9. $1 - x^4$; $(1+x^2)(1+x)^2$, $(1+x^2)(1-x)^2$.
11. $10\left(1 + \frac{13}{1000}\right)^{\frac{1}{3}}$.

Pure Mathematics.

1. $3 + \frac{1}{3+}\,\frac{1}{6+}\,\frac{1}{3+}\,\frac{1}{6}$, &c.; $\frac{1}{1+}\,\frac{1}{2+}\,\frac{1}{6+}\,\frac{1}{3+}\,\frac{1}{6}$, &c.; write x for the value of the continued fraction; thus $x = \frac{1}{2-x}$; therefore $x = 1$.
2. 35, 0; 24, 7; 13, 14; 2, 21. 4. 1·25893. 5. $\frac{3}{8}$.
7. 3, $\frac{7}{9}$, $\frac{1}{3}$. 9. 4, 4, -3. 10. $x + 3y = 0$.
11. $yy' = 2a(x+x')$; to determine the points of intersection of this tangent with the parabola, $y^2 = 4a'x$,

we have the quadratic, $y^2 - \frac{2a'y'}{a}.y + \frac{a'y'^2}{a} = 0$; if y_1, y_2 be the roots of this equation, we then have $\frac{2y_1y_2}{y_1+y_2} = y'$.

B. $\pm 1, \pm\sqrt{(-1)}, \pm\frac{1}{\sqrt{(2)}}\{1+\sqrt{(-1)}\}, \pm\frac{1}{\sqrt{(2)}}\{1-\sqrt{(-1)}\}$.

C. Use *For.* p. 98, l. 2, and p. 99, last line.

D. $\sqrt{\left(1-\frac{a}{b}\right)}$, where $a = \frac{1}{2}[A+C+\sqrt{\{B^2+(A-C)^2\}}]$ and $b = \frac{1}{2}[A+C-\sqrt{\{B^2+(A-C)^2\}}]$.

Mixed Mathematics.

1. Diminished.

3. The tension of the string, that has its point of attachment shifted, is double that of each of the others.

4. $\tan^{-1}\mu, \frac{\mu W}{\sqrt{(1+\mu^2)}}$.

6. $\sqrt{(2ga)}$. 7. $2g$. 10. $\frac{17}{64}$ths.; $\frac{1}{8}$th.

11. The action of the pump is assisted. *A.* With the same notation as in *A*, p. 25, $(R-Q)\,P - RQ = 0$.

B. Tait and Steele's *Part. Dyn.* Cap. V., Art. 144.

C. Besant's *Hydromech.*, Cap. II.

Pure Mathematics.

2. $\pi - \cos^{-1}\frac{1}{3} = 2\cos^{-1}\frac{1}{\sqrt{3}}$.

5. (1) $\pm\frac{bx}{a\sqrt{(a^2-x^2)}}$; (2) $\frac{1}{\sqrt{(2ax+x^2)}}$; (3) $\frac{1}{\sqrt{(1-x^2)}}$; (4) $\frac{6}{x}$.

6. $1 - \frac{nx}{2} + \frac{n(n+1)x^2}{8}$. 7. (1) $\frac{1}{2}$; (2) $-\frac{1}{2}$.

8. Radius of base $= \frac{1}{\sqrt{2}}$ (rad. of hemisphere).

12. (1) $\log\sqrt{\left(\frac{x-5}{x-3}\right)}$; (2) $\frac{a^2}{2}\sin^{-1}\frac{x}{a} - \frac{x\sqrt{(a^2-x^2)}}{2}$; (3) $\text{vers}^{-1}\frac{x}{a}$. 13. $\frac{C}{\sqrt{(A^2+B^2+C^2)}}$.

MARINES, AUGUST, 1873. PP. 42–51.

Arithmetic and Logarithms.

1. $2\frac{83}{120}$. 2. $1\frac{17}{247}$. 3. $82\frac{59}{88}$.
4. $2\frac{4}{51}$. 5. 17·20779. 6. 2·0577.
7. 1·0368. 8. 3·080702. 9. ·000475.
10. $16\frac{13}{45}$. 11. $\frac{13}{210}$. 12. $1\frac{19}{245}$.
13. $\frac{87}{88}$. 14. $\frac{125}{336}$. 15. 11011.
16. ·00423. 17. $\frac{193}{200}$. 18. $\cdot 0\dot{7}1428\dot{5}$.
19. 75081·6. 20. £3. 19*s*. $1\frac{1}{5}$*d*. 21. ·0028978....
22. £13. 23. £304. 24. £354. 13*s*. 4*d*.
25. 7690. 26. ·313026. 27. £61. 13*s*. 1*d*.
28. 143171·33. 29. £37. 9*s*. $3\frac{9}{10}$*d*.

Euclid.

2. 325 sq. ft.

Algebra.

1. 278. 2. -16. 3. $a+b-\sqrt{(ab)}$.
4. $x^4+5x^3-x^2-5x$. 5. $\frac{m-2}{m+2}$.
6. (1) 2; (2) $2\frac{1}{3}$, $-1\frac{7}{9}$, $-1\frac{1}{9}$; (3) 3, $-8\frac{1}{3}$.
7. 60. 8. 12, 8. 9. 32.
11. -36.

Mathematics. Sec. I., 4th Paper.

1. 0, 0.
2. Solving the equation, $\frac{n}{2}\left(2-\frac{n-1}{5}\right)=4$, we get n an impossible quantity.
3. The coeff. of the 6th term; the first term.
4. 250° 31′ 44″. 5. $n\pi+\frac{\pi}{4}$. 6. 89° 59′ 35″.
7. 76° 11′ 16″. 8. 206·783 yds. 9. *For*. p. 97 (V).
10. 65·958 c. yds.

Mathematics. Sec. II.

1. 52° 32′ 56″ 2. $\frac{2500\pi}{3}$ sq. ft.
3. $bc=ad$; $\frac{c(b\sim d)}{\sqrt{(c^2+d^2)}}$.

4. $yy' = 2a(x + x' + 2a)$; 0, $2a$.

6. (1) $\dfrac{-2a}{3(a-x)^{\frac{1}{3}}}$; (2) $\dfrac{x^4 + 3a^2x^2 - 2a^3x}{(a^2+x^2)^2}$;

(3) $4\sec^2 2x - 2\operatorname{cosec}^2 x$; $a^x\{\cos x[(\log a)^2 - 1] - 2\sin x \log a\}$.

7. $$\sin 2\theta = 2\theta - \frac{(2\theta)^3}{\lfloor\underline{3}} + \frac{(2\theta)^5}{\lfloor\underline{5}} \ldots$$
$$+ \frac{(2\theta)^n}{\lfloor\underline{n}} \sin\frac{n\pi}{2} + \frac{(2\theta)^{n+1}}{\lfloor\underline{n+1}} \sin\left[\frac{n+1}{2}\pi + \theta_1(2\theta)\right].$$

8. $X(x^2 - ay) + Y(y^2 - ax) - axy = 0$; $2^{\frac{1}{3}}a$, $2^{\frac{2}{3}}a$.

9. (1) $\frac{2}{15}(3x - 2a)(a+x)^{\frac{3}{2}}$;

(2) $\dfrac{1}{2\sqrt{2}} \cdot \log\cot\left(\dfrac{x}{2} - \dfrac{\pi}{8}\right)\cot\left(\dfrac{x}{2} + \dfrac{\pi}{8}\right)$; let $x = 1 - y$,

thus $\int\sqrt{(2x - x^2)}\,dx = -\int\sqrt{(1-y^2)}\,dy$, a known form; $\dfrac{\pi}{4}$.

11. $\frac{4}{3}\pi a^2 b$.

Mixed Mathematics. Sec. III.

3. 8*s*. per lb.
4. $\frac{5}{8}$ (altitude of prism).
6. $\dfrac{144}{g}$ lbs.
7. 32.
9. $3\frac{11}{288}$ lbs.; $3\frac{29}{32}$ lbs.
10. $426\frac{2}{13}$ c. in.
11. $\dfrac{17280\pi}{h}$ lbs., where h is the height in inches of the water-barometer.

COOPER'S HILL, JULY, 1873. PP. 52–71.

Arithmetic and Mensuration.

1. $9\frac{203}{240}$.
2. $1\frac{47}{51}$.
3. $\frac{5}{256}$.
4. $\frac{3}{4}$.
5. 4752·131585.
6. 97.38773.
7. 604·1652.
8. 79·3065.
9. ·01875.
10. $22\frac{9}{35}$.
11. $65\frac{33}{40}$.
12. 546.
13. $1\frac{16}{27}$.
14. 979·08 ft.
15. 52·28 hrs. = 2 d. 4 hr. 16′ 48″.
16. 27·6022.
17. ·9638.
18. $\cdot 01597\dot{2}$.
19. £518. 6*s*. $1\frac{1}{4}$*d*.
20. 125 ac. 1 r. 25 p. 7 sq. yds. 4 sq. ft. 108 sq. in.

21. 38 ac. 22. £3. 14*s*. 0*d*.
23. 133 c. ft. 1677$\frac{1}{4}$ c. in. 24. 1$\frac{1}{24}$ ft.
25. 4502 lks. 26. 8448000 gals. 27. $\frac{7695\pi}{2}$ c. in.
28. 150 lbs. 29. 2 ac. 1 rd. 38·37 perches.

Euclid, Algebra, and Trigonometry (1).

2. On the given straight line describe an equilateral triangle. Bisect the angles at the base, and from the point in which the bisectors meet draw parallels to the sides of the triangle.
4. See Woolwich, May, p. 17.
5. $3(b-c)(2a-b+c)$; $(x-11)(x-8)$; $(x^2+xy+y^2)(x^2-xy+y^2)$.
6. Incorrect. 7. $x-2$; $(a-c)(ad-bc)+(b-d)^2=0$.
8. (1) $\frac{a+b+c}{3}$; (2) $7\frac{1}{4}$, $4\frac{3}{4}$;

(3) $$\frac{(a+b+c)(b+c+2a)}{2}, \quad \frac{(a+b+c)(a+c+2b)}{2},$$
$$\frac{(a+b+c)(a+b+2c)}{2}.$$

9. $\frac{n}{2}(3n-1)$; $\left\{\frac{n(n+1)}{2}\right\}^2$; $\frac{n(n-1)(n+1)(3n+2)}{12}$.
11. *For.* p. 88.
13. Area $= rs = Rr(\sin A + \sin B + \sin C)$ by *For.* p. 98 (i); *For.* p. 100.

Euclid, Algebra, and Trigonometry (2).

1. The third side always touches the escribed circle opposite to the angle included by the two sides given in position; see *For.* p. 96 (ii).
3. If A, A' be the centres of the given circles, P any position of the point, PT, PT' tangents to the respective circles, we have the triangles APT, $A'PT'$ similar; therefore $AP : A'P :: AT : A'T'$, a fixed ratio; therefore, by Todhunter's *Euc.*, App. Art. 55, the locus of P is a circle.
5. 45·33003; 156·62111. 6. $n(n-1)$; $\frac{\underline{|n}}{\underline{|n-r}}$.

7. $\dfrac{\lfloor\underline{n}}{\lfloor\underline{r}\,\lfloor\underline{n-r}}$; $\dfrac{n(n+1)\ldots(n+r-1)}{\lfloor\underline{r}}$; $r+(-\frac{1}{2})^{r+1}$; $r-1$.

8. $\frac{5}{36}$; 7 to 5.

9. *For*. p. 56; excluding zero values 8, 1; 3, 3; 2, 4.

11. (2) $\cot\alpha - 2^n \tan 2^n\alpha$.

Statics.

2. 12 in. from the weight; 27 oz.

4. A the point from which W is suspended, D_1, D_2..., the different marks of graduation; then $AD_1 = 5$ in., $AD_2 = 8$ in., $AD_3 = 11$ in. &c.

5. 192 lbs. 6. 36 lbs.

Dynamics.

1. $\dfrac{7h}{16}$ from the lower end. 2. 320 ft. per sec.; 30°.

3. $\frac{5}{6}g$ lbs.

4. The ball ($3m$) is brought to rest; the ball ($5m$) is made to move with a velocity of 18 ft. per sec.

5. $(864)^2 : (867)^2$. 6. $\dfrac{30}{\pi}$.

Pure Mathematics (1).

1. 5; $\pi - \tan^{-1}\frac{3}{4}$. 4. $y = a$; $x = 0$.

10. (1) The origin; (2) impossible; (3) straight line in the plane of yz, parallel to the axis of z, at the distance a from it; (4) two planes parallel to yz.

12 and 13. See Woolwich, May, p. 27. A. The ellipse, $4x^2 + y^2 = 2ax$. B. Take the shortest distance between the two given lines as the axis of z, and its middle point as the origin; take the lines bisecting the angles between the projections of the given lines on the plane to which z is at right angles as the other axes; then the equations of the given lines are

$$z = c,\ y = mx;\quad z = -c,\ y = -mx;$$

therefore by the problem,

$$(z+c)^2 + \frac{(y+mx)^2}{1+m^2} - (z-c)^2 - \frac{(y-mx)^2}{1+m^2} = k^2;$$

therefore $cz(1+m^2) + mxy = \dfrac{k^2(1+m^2)}{4}$,

the equation to a hyperbolic paraboloid.

Pure Mathematics (2).

2. Woolwich, Sept., p. 39. 3. 120°.
4. 62° 8′ 52″; 50° 12′ 58″.
5. (1) $\frac{1}{\sqrt{(1+x^2)}}$; (2) $\tan^4 x$; (3) $\frac{2}{1+x^2}$.
7. The triangle is a maximum, when it is equilateral; use $A = \frac{1}{2}b^2 \sin\theta = 2a^2 \sin\theta \cos^2\frac{\theta}{2}$, where a is the radius of the circle, and θ the vertical angle of the isosceles triangle.
8. $-\frac{dx'}{dy'}$; if x is the abscissa of one extremity of the focal chord, $\frac{a^2}{x}$ is that of the other extremity.
9. The witch; $x = \frac{3a}{2}$, $y = \pm\frac{2a}{\sqrt{3}}$.
10. $p = \frac{r}{\sqrt{2}}$, $\phi = \frac{\pi}{4}$.
11. Todhunter's *Diff. Cal.* Art. 332; using $p = \frac{r}{\sqrt{2}}$, we get $p' = \frac{r'}{\sqrt{2}}$.
12. (1) $\log\frac{a+x}{a-x}$;

(2) $\int\frac{x^2dx}{\sqrt{(a^2-x^2)}} = \int\frac{a^2dx}{\sqrt{(a^2-x^2)}} - \int\sqrt{(a^2-x^2)}dx$, known forms; (3) $\tan x - x$. *A.* $\frac{b^2}{a}$, $\frac{a^2}{b}$.

B. Epicycloid, $\frac{8b(a+b)}{a}$; hypocycloid, $\frac{8b(a-b)}{a}$.

Mixed Mathematics.

1. Todhunter's *Analytical Statics*, Art. 50.
2. Take α the inclination of the plane, β that of the rod, θ that of the action at A to the horizon; Q the action at A, R the resistance of the plane; G the centre of gravity of the rod, $AG = a$, $BG = b$.

Taking moments about A, we have

$$W.a \sin\beta = R(a+b)\cos(\alpha-\beta) \;\ldots\ldots\; (1).$$

Resolving vertically and horizontally,

$$Q\sin\theta = W - R\cos\alpha \quad\text{..............}\quad (2),$$

$$Q\cos\theta = R\sin\alpha \quad\text{....................}\quad (3).$$

The equations (1), (2), (3) determine Q and θ.

3. $\bar{x}=0$, $\bar{y}=\dfrac{4a}{3\pi}$; $\bar{x}=0$, $\bar{y}=\dfrac{3a}{2\pi}$.

4. Let $2l$ = length of chain, $2a$ = distance between the fixed points, c = length of chain whose weight is equal to the tension at the lowest point. Then the tension at either point of support is a minimum when it equals the weight of a length $\dfrac{al}{c}$ of the chain, and l and c are determined by

$$l=\frac{c}{2}\left(e^{\frac{a}{c}}-e^{-\frac{a}{c}}\right),\quad (l^2+c^2)\,c^2=a^2l^2.$$

6. Pressure at vertex

$$=\frac{mv^2}{\rho}+mg=m\left(\frac{2g.2a}{4a}+g\right)=2mg.$$

8. Tait and Steele's *Part. Dyn.*, Cap. VIII., Ex. 8.
10. 3 : 2.
11. Besant's *Hydrost.*, Art. 61, Ex. 4.
12. Increase. 13. $8(1+100\alpha):1$.

B. O the centre of the circle, P an element of the string, S the middle point of the given radius; produce SO to meet the circle at K. Let $\angle PSO=\theta$, $\angle POK=\phi$, $SP=r$, $OP=a$; then

$$T=\frac{ma^2}{2}(1+\cos\phi);$$

$$P=\frac{ma}{2}(1+\cos\phi)+mr\cos(\phi-\theta).$$

C. An ellipse.

D. Let $OBED$ be the square, O the angular point beneath the surface, AC the line in which the surface of the liquid cuts the plane of the lamina; $OA=x$, $OC=y$, $OB=a$, ρ = density of liquid, σ = density of lamina. When OE is perpendicular to AC, $x=a\sqrt{\left(\dfrac{2\sigma}{\rho}\right)}$; when OE is not perpen-

Euclid.

3. Draw OD, OE perpendiculars to CA, CB. Then from the triangles OAD, OBE, we have $OD = OE$. Also $ODCE$ is a rectangle, and therefore a square. Hence CO bisects the angle ACB.
4. With the fig. of Todhunter's *Euc.* II. 14, the sq. on GH has the same perimeter as the rect. BD, and $GH > HE$.
6. Let BA, QA be produced to C, R; then
   $$\angle BPQ = \angle BAQ = \angle CAR = \angle CAP = \angle BQP.$$
8. *For.* p. 22, l. 11.
10. Join OB, OC. Then the angle BOP is common to the two triangles BOP, DOB, and
    $$OP : OB :: OB : OD;$$
    therefore $\angle OBP = \angle ODB$. Similarly, $\angle OCP = \angle ODC$. Otherwise, describe a circle about the triangle BPD. Then, because the rect. OP, $OD =$ the sq. on OB; therefore OB touches the circle, and the angle $OBP =$ the angle BDP in the alternate segment.

Algebra.

2. $x^9 - 12x^7 + 4x^6 + 33x^5 - 39x^4 - 210x^3 - 166x^2 - 286x - 312$;
   $3(a+b+c)(5b+7c-a)$; $(x-7)(x-12)$.
3. (β). Reduce the second side, or the first side
   $$= xy(x^2-y^2) + yz(y^2-z^2) + zx(z^2-x^2)$$
   $$= xy(x+y)(x-y) + yz(y+z)(y-z) + zx(z+x)(z-x)$$
   $$= (x+y+z)\{xy(x-y) + yz(y-z) + zx(z-x)\}$$
   = the second side.
4. $x^2 - 29x + 23$; $(x^2-29x+23)^2(x^2+29x-27)$.
5. *For.* p. 37; each $= \dfrac{(y+z)+(z+x)-(x+y)}{(b+c)+(c+a)-(a+b)} = \dfrac{z}{c}$;
   similarly, each $= \dfrac{x}{a}$ or $\dfrac{y}{b}$.
6. (α) 6; (β) $5\frac{1}{2}$, $3\frac{1}{4}$; (γ) 2, $-3\frac{16}{17}$;
   (δ) $x = 8$ or 6, $y = 3$ or 4.
7. $22\frac{1}{2}$, $7\frac{1}{2}$. 9. 2; $\pm 2(1-\sqrt{3})$. 10. 6.
11. *For.* pp. 67–69; $p^2 - 4q = 0$; 2 or $-\frac{1}{4}$.
12. $1 + 3x + 6x^2 + 10x^3$, $\frac{1}{2}(r+1)(r+2)$;
    $$1 + x + \frac{3x^2}{2} + \frac{5x^3}{2}, \quad \frac{1.3.5\ldots(2r-1)}{\underline{|r}}.$$

Pure Mathematics.

1. $\frac{13775}{2651}$; 3.

2. $$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\dots \text{ to } n \text{ factors}$$

$$=\frac{2^2-1}{2^2}\cdot\frac{3^2-1}{3^2}\cdot\frac{4^2-1}{4^2}\dots\frac{n^2-1}{n^2}\cdot\frac{(n+1)^2-1}{(n+1)^2}$$

$$=\frac{(1.3)(2.4)(3.5)(4.6)\dots\{(n-1)(n+1)\}\{n(n+2)\}}{2^2.3^2.4^2\dots n^2(n+1)^2}$$

$$=\frac{n+2}{2(n+1)}.$$

4. $\frac{1}{6}$, $\frac{1}{3}$. 5. $\dfrac{1}{x-a+b}-\dfrac{2}{x-a}+\dfrac{1}{x-a-b}$.

6. Todhunter's *Pl. Trig.*, Art. 326.

7. $B=54° \, 34' \, 3''\cdot87$, $C=83° \, 25' \, 56''\cdot13$.

8. $B=75°$, $C=45°$, $a=\sqrt{(6)}$.

10. $\pm\sqrt{(-2)}$, $-\frac{1}{2}\pm\dfrac{\sqrt{5}}{2}$. 11. $x+3y=0$.

A. $$\lfloor\underline{20}\left\{\frac{-3}{\lfloor\underline{17}\lfloor\underline{2}}+\frac{(-3)^2 4}{\lfloor\underline{16}\lfloor\underline{2}}+\frac{4^4}{\lfloor\underline{16}\lfloor\underline{4}}+\frac{(-3)^3 4^2}{\lfloor\underline{15}\lfloor\underline{3}\lfloor\underline{2}}+\frac{(-3)^6}{\lfloor\underline{14}\lfloor\underline{6}}\right\}.$$

B. We have $e^x.e^{y\sqrt{(-1)}}=a+b\sqrt{(-1)}$; substitute for $e^{y\sqrt{(-1)}}$ its value in terms of $\cos y$ and $\sin y$, and equate the rational and surd parts; thus $x=\frac{1}{2}\log(a^2+b^2)$, $y=\tan^{-1}\dfrac{b}{a}$.

C. Draw *AD*, *BE*, *CF* perpendiculars to the sides *BC*, *CA*, *AB* of the triangle *ABC*. By the use of *For.*, p. 97, it is easily shewn that the triangle *DEF* has its sides respectively equal to $a \cos A$, $b \cos B$, $c \cos C$. Let R and R' be the radii of the circumscribing circles of the triangles *ABC*, *DEF*; then, by *For.*, p. 20, $R=2R'$. Now,

$$\Delta\, DEF=\frac{def}{4R'};$$

and $$\Delta\, ABC=\frac{abc}{4R};$$

therefore $\Delta\, DEF : \Delta\, ABC :: R.def : R'.abc$

$$:: 2^* \cos A \cos B \cos C : 1.$$

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* The figure 2 is wanting in the original paper.

Mixed Mathematics.

3. The strings are at right angles. 5. 9·66 ft. per sec.

6. 32·2; 38640; $\frac{1250}{g}$ ft.; $\frac{25}{g}(2 \mp \sqrt{2})$ sec. after projection.

8. $\frac{3g}{11}$ ft. per sec.; $\frac{50g}{11}$ lbs. 11. 74⅔ yds.

Pure Mathematics.

4. $9a^2 - 52ax + 72x^2$;

$$-\frac{\cos x}{\sin^2 x}; \quad \frac{1}{\sqrt{(1-x^2)}}; \quad \frac{2a^2}{(a^2-2x^2)(a^2-x^2)^{\frac{1}{2}}}; \quad \frac{2}{x \sin 2 (\log x)}.$$

5. $x^2 \frac{d^{n+2}y}{dx^{n+2}} + (2n-1)\, x \frac{d^{n+1}y}{dx^{n+1}} + (n-1)^2 \frac{d^n y}{dx^n}$.

6. $(a^2+b^2)^{\frac{1}{2}} \sin\phi . x + (a^2+b^2) \sin 2\phi \frac{x^2}{\lfloor\underline{2}}$

$$+ (a^2+b^2)^{\frac{3}{2}} \sin 3\phi . \frac{x^3}{\lfloor\underline{3}} + \ldots + (a^2+b^2)^{\frac{r}{2}} \sin r\phi . \frac{x^r}{\lfloor\underline{r}} + \ldots.$$

7. No; a maximum when $x = 2$, a minimum when $x = 3$.

8. We get $\tan\phi = -\cot 2\theta$.

9. $x - 1 = 0$; $r \sin(\alpha - \theta) = a\alpha$. 10. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

11. $-\operatorname{cosec} x$; $\frac{1}{2a} \log \frac{x-a}{x+a}$; $\frac{1}{a} \log \frac{x-2a}{x-a}$.

12. a^2. 13. $\frac{a}{3}, \frac{b}{3}, \frac{c}{3}$;

$$\frac{\frac{1}{a}}{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}, \quad \frac{\frac{1}{b}}{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}, \quad \frac{\frac{1}{c}}{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}};$$

$$\frac{3}{\sqrt{(a^2+b^2+c^2)} \sqrt{\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)}}.$$

14. $\pi(c^2-\gamma^2)$, $\pi(c^2-\alpha^2)$, $\pi(c^2-\beta^2)$.

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Fig. 1

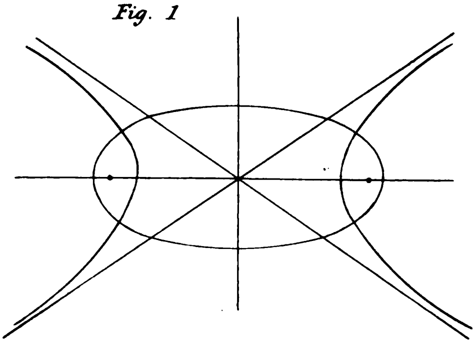


Fig. 2

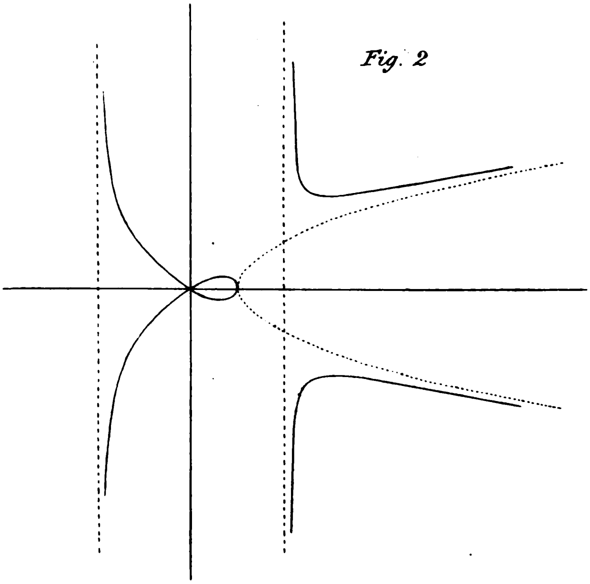
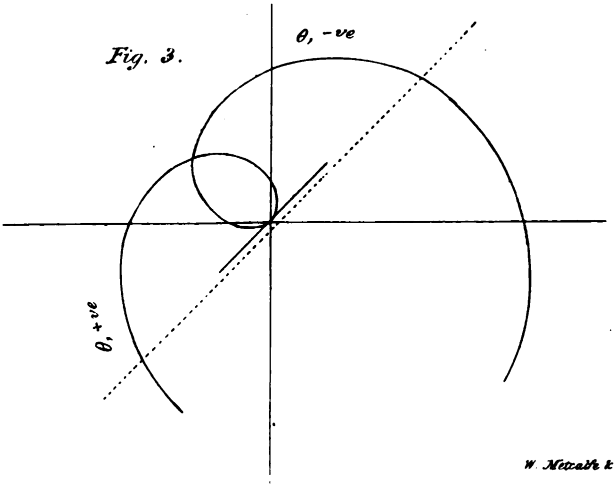


Fig. 3.



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